



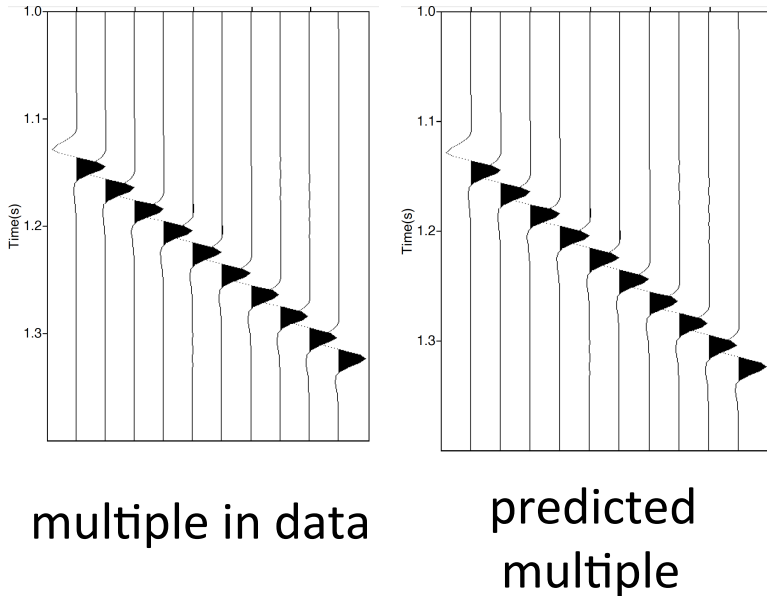
The internal-multiple *elimination* algorithm for all reflectors for 1D earth Part 1: strengths and limitations

Yanglei Zou* and Arthur B. Weglein

May 29th, 2014
Austin, TX

ISS internal multiple elimination algorithm

Case 1



Solution

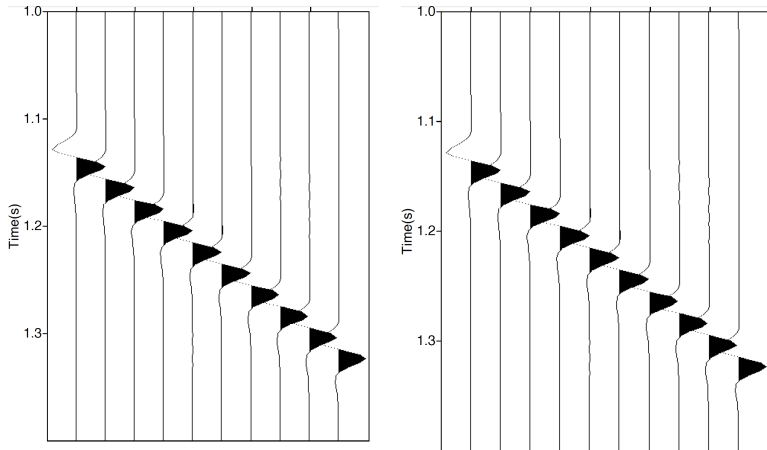
ISS Internal-multiple attenuation
algorithm

+ adaptive subtraction

(Figure from Jinlong Yang M-OSRP annual report 2013)

ISS internal multiple elimination algorithm

Case 1



multiple in data

predicted
multiple

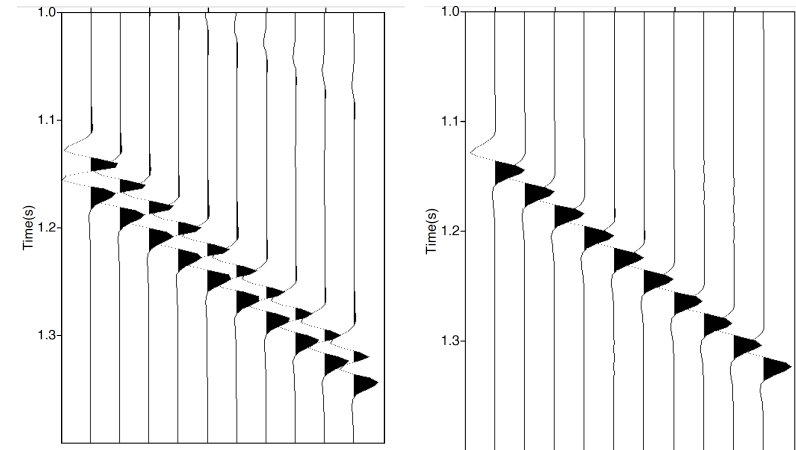
Solution

ISS Internal-multiple attenuation
algorithm

+ adaptive subtraction

(Figure from Jinlong Yang M-OSRP annual report 2013)

Case 2



primary and multiple
interfering in data

predicted
multiple

Solution

Three-pronged strategy
for internal multiple removal
Weglein (2013)

The three-pronged strategy

The three-pronged strategy

1. Pre-requisites for on-shore application;

The three-pronged strategy

- 1. Pre-requisites for on-shore application;**
- 2. Beyond ISS internal multiple attenuation;**

The three-pronged strategy

- 1. Pre-requisites for on-shore application;**
- 2. Beyond ISS internal multiple attenuation;**
 - a) Removing artifacts/spurious events**
 - b) Internal multiple elimination**

The three-pronged strategy

- 1. Pre-requisites for on-shore application;**
- 2. Beyond ISS internal multiple attenuation;**
 - a) Removing artifacts/spurious events**
 - b) Internal multiple elimination**
- 3. New adaptive subtraction criteria.**

The three-pronged strategy

- 1. Pre-requisites for on-shore application;**
- 2. Beyond ISS internal multiple attenuation;**
 - a) Removing artifacts/spurious events**
 - b) Internal multiple elimination**
- 3. New adaptive subtraction criteria.**

Beyond ISS internal multiple attenuation

Internal-multiple attenuation
algorithm

Araújo et al.(1994),Weglein et al.(1997)

Beyond ISS internal multiple attenuation

Internal-multiple attenuation
algorithm

Araújo et al.(1994),Weglein et al.(1997)

Predicting correct amplitude

Beyond ISS internal multiple attenuation

Internal-multiple attenuation
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Early ideas
Ramírez(2005)

Beyond ISS internal multiple attenuation

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Early ideas

Ramírez(2005)

Elimination of first order
internal multiples generated
at the shallowest reflector
(1D normal incidence)

Herrera(2012)

Beyond ISS internal multiple attenuation

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Elimination of first order internal multiples generated at the shallowest reflector (1D normal incidence)

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The internal-multiple elimination algorithm for all reflectors for 1D earth offset data

(This presentation)

The structure of this presentation

ISS internal multiple
attenuation algorithm

```
graph TD; A[ISS internal multiple attenuation algorithm] --> B[Attenuation Factor]; B --> C[ISS internal multiple elimination algorithm];
```

Attenuation Factor

ISS internal multiple
elimination algorithm

The structure of this presentation

ISS internal multiple
attenuation algorithm

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graph TD; A[ISS internal multiple attenuation algorithm] --> B[Attenuation Factor]; B --> C[ISS internal multiple elimination algorithm];
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Attenuation Factor

ISS internal multiple
elimination algorithm

ISS internal multiple attenuation algorithm

The 1D normal incidence version of the internal multiple attenuation algorithm is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

ISS internal multiple attenuation algorithm

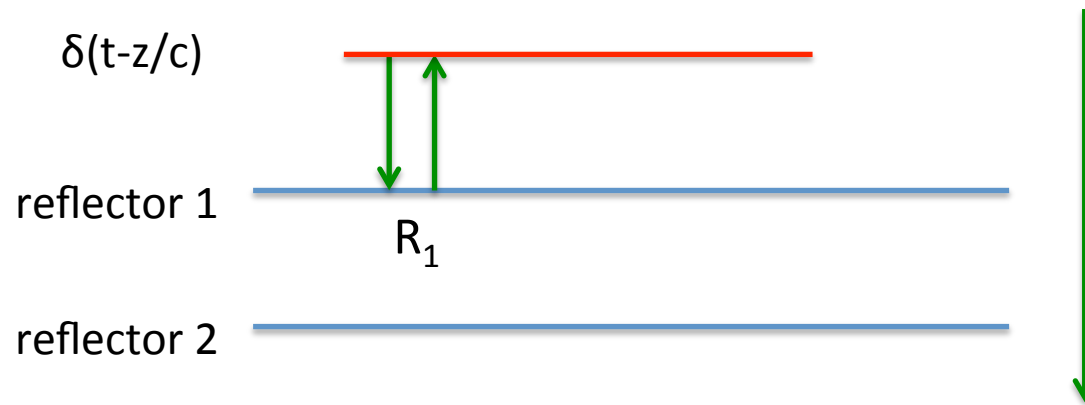
reflector 1 

reflector 2 

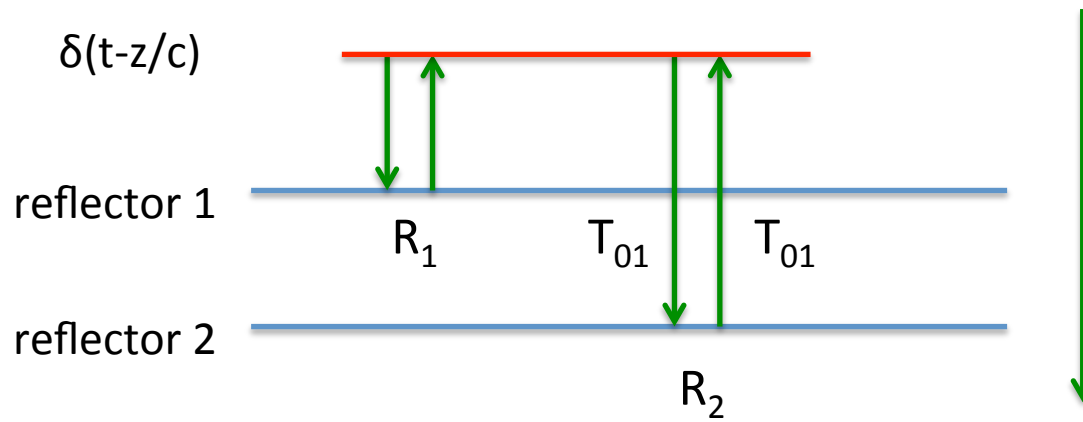
ISS internal multiple attenuation algorithm



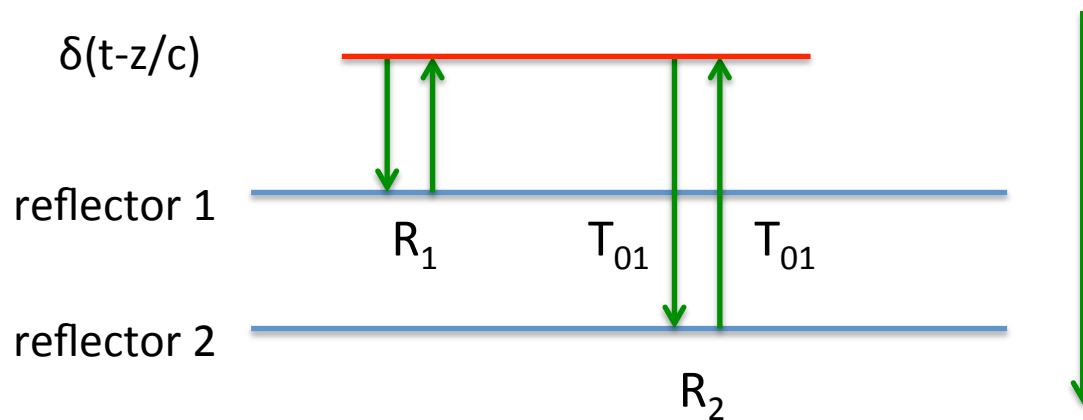
ISS internal multiple attenuation algorithm



ISS internal multiple attenuation algorithm



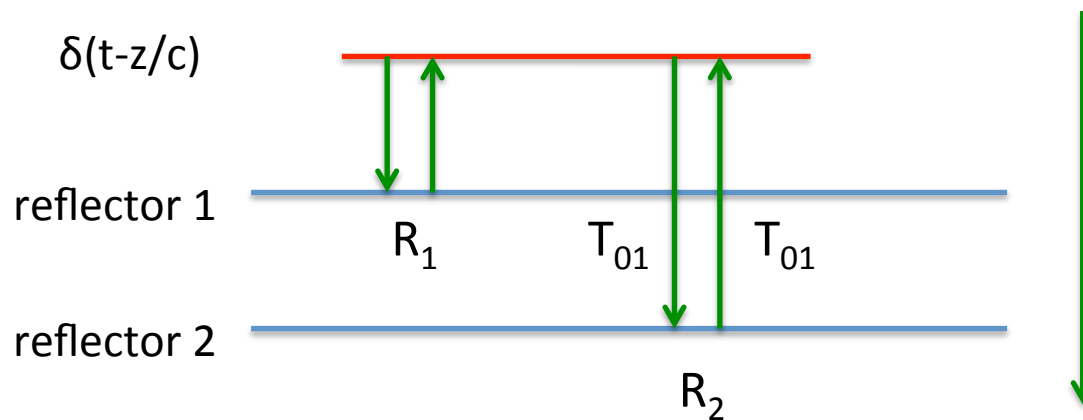
ISS internal multiple attenuation algorithm



The reflection data caused by an impulsive incident wave $\delta(t-z/c)$ is:

$$D(t) = R_1\delta(t - t_1) + T_{01}R_2T_{10}\delta(t - t_2) + \dots$$

ISS internal multiple attenuation algorithm



The reflection data caused by an impulsive incident wave $\delta(t-z/c)$ is:

$$D(t) = R_1\delta(t - t_1) + T_{01}R_2T_{10}\delta(t - t_2) + \dots$$

Make a water speed migration of $D(t)$ with $k_z = \frac{2\omega}{c_0}$
and pseudo-depths: $z_1 = \frac{c_0 t_1}{2}$ $z_2 = \frac{c_0 t_2}{2}$.

$$b_1(z) = R_1\delta(z - z_1) + T_{01}R_2T_{10}\delta(z - z_2) + \dots$$

ISS internal multiple attenuation algorithm

The ISS internal-multiple attenuation algorithm prediction is:

$$b_3(t) = R_1 R_2^2 T_{01}^2 T_{10}^2 \delta(t - (2t_2 - t_1))$$

ISS internal multiple attenuation algorithm

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$$b_3(t) = R_1 R_2^2 T_{01}^2 T_{10}^2 \delta(t - (2t_2 - t_1))$$

The actual first order internal multiple is:

$$-R_1 R_2^2 T_{01} T_{10} \delta(t - (2t_2 - t_1))$$

ISS internal multiple attenuation algorithm

The ISS internal-multiple attenuation algorithm prediction is:

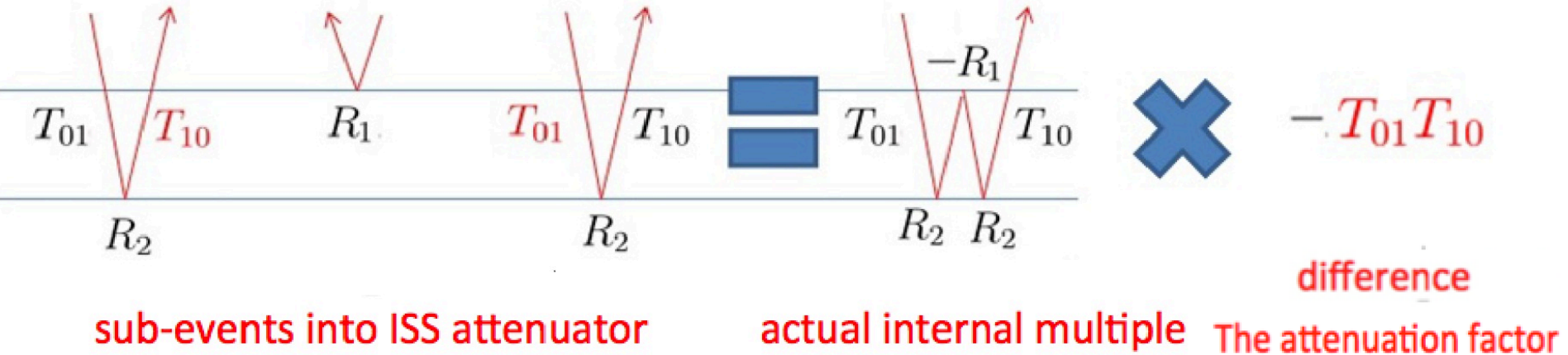
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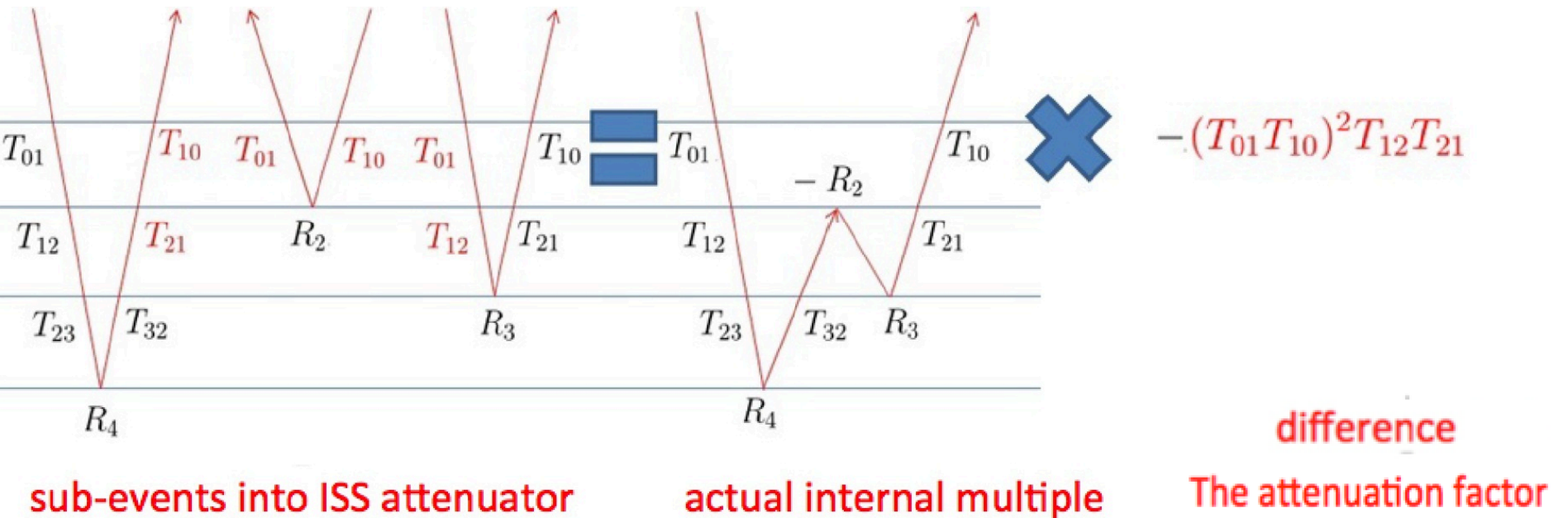
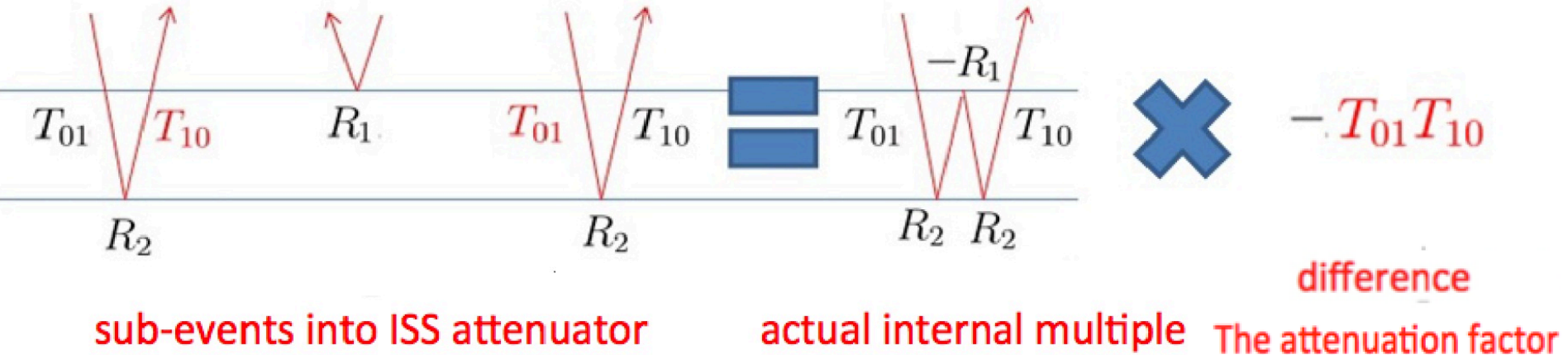
$$-R_1 R_2^2 T_{01} T_{10} \delta(t - (2t_2 - t_1))$$

The time prediction is precise, and the amplitude of the prediction has an extra power of $T_{01} T_{10}$ which is called the **attenuation factor**. This factor is first noted by Weglein and Matson (1998)

The attenuation factor (1D normal incidence)



The attenuation factor (1D normal incidence)



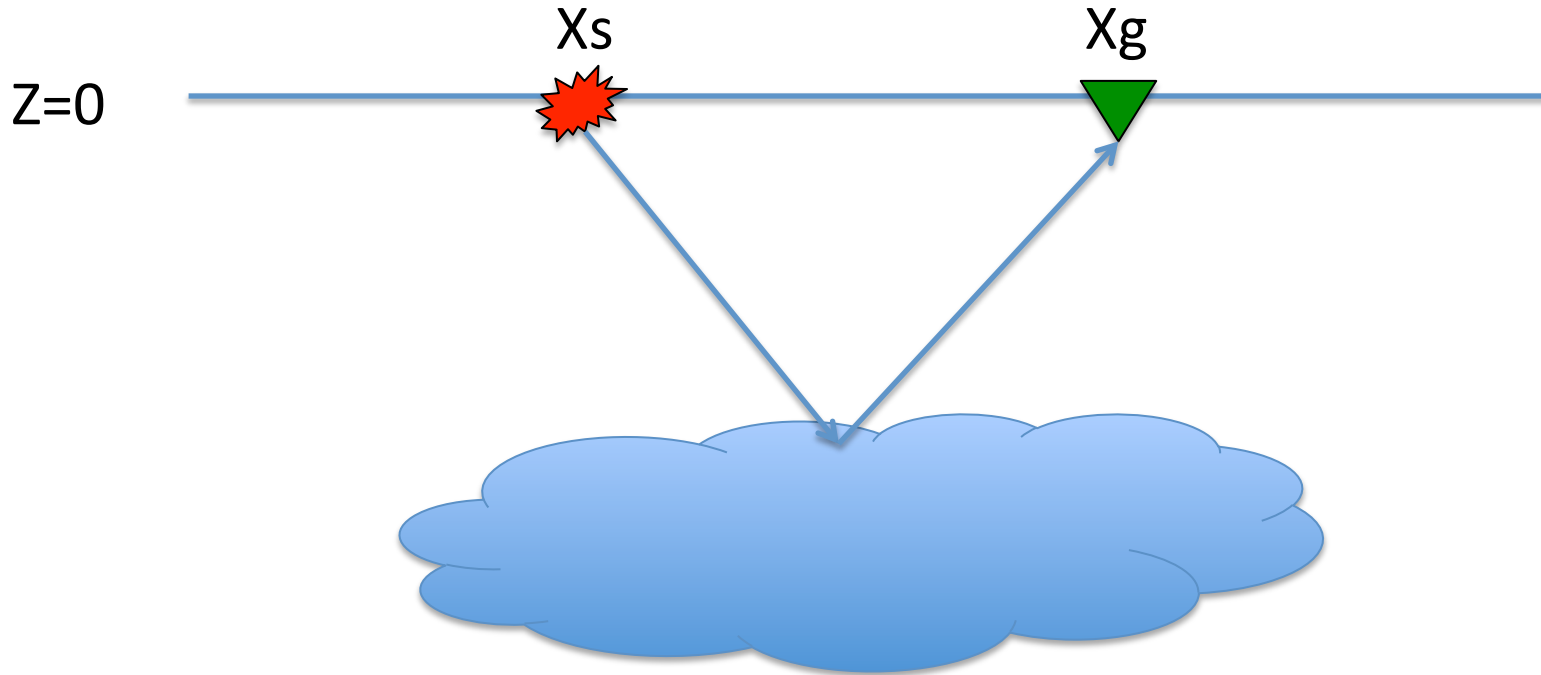
The attenuation factor (1D normal incidence)

A generalization of the **attenuation factor(AF)** is:
(1D normal incidence)

$$AF_j = \begin{cases} T_{0,1}T_{1,0} & (j = 1) \\ \prod_{i=1}^{N-1} (T_{i-1,i}^2 T_{i,i-1}^2) T_{j,j-1} T_{j-1,j} & (1 < j < J) \end{cases}$$

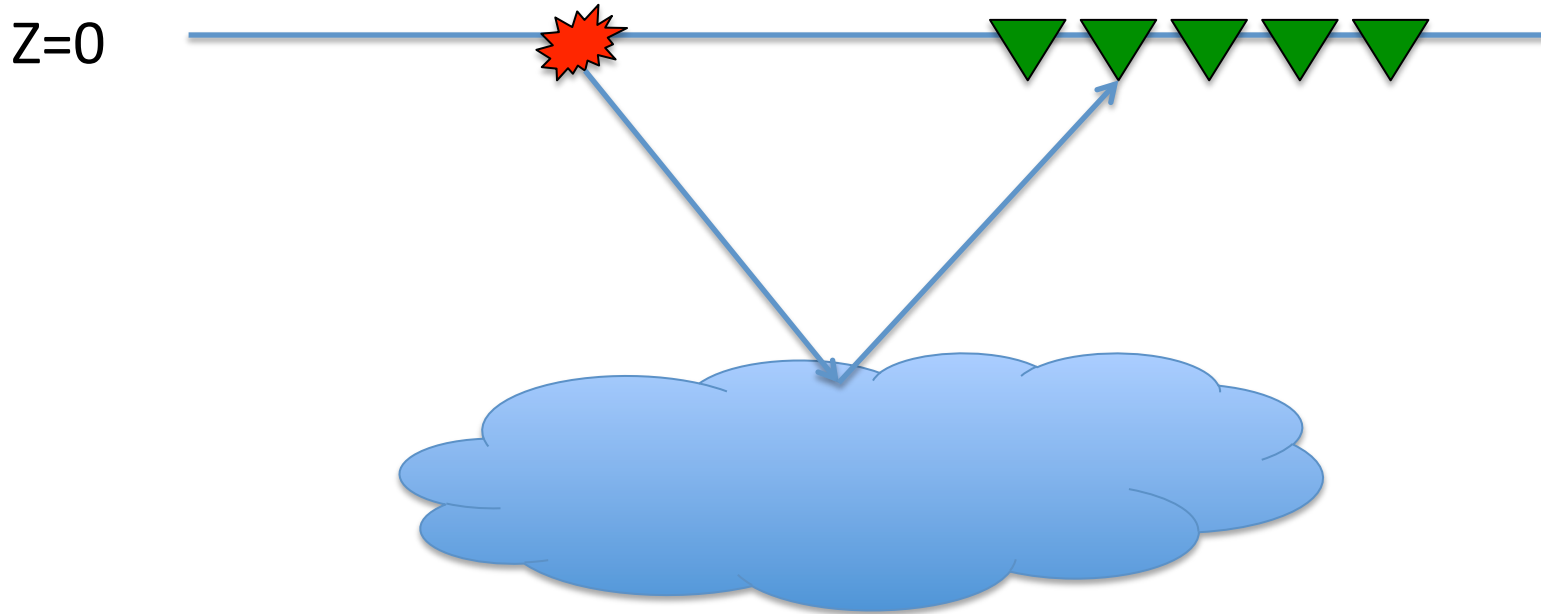
AF_j is the attenuation factor for first order internal multiples with a downward reflection at the j^{th} reflector.

The attenuation factor (1D earth offset data)



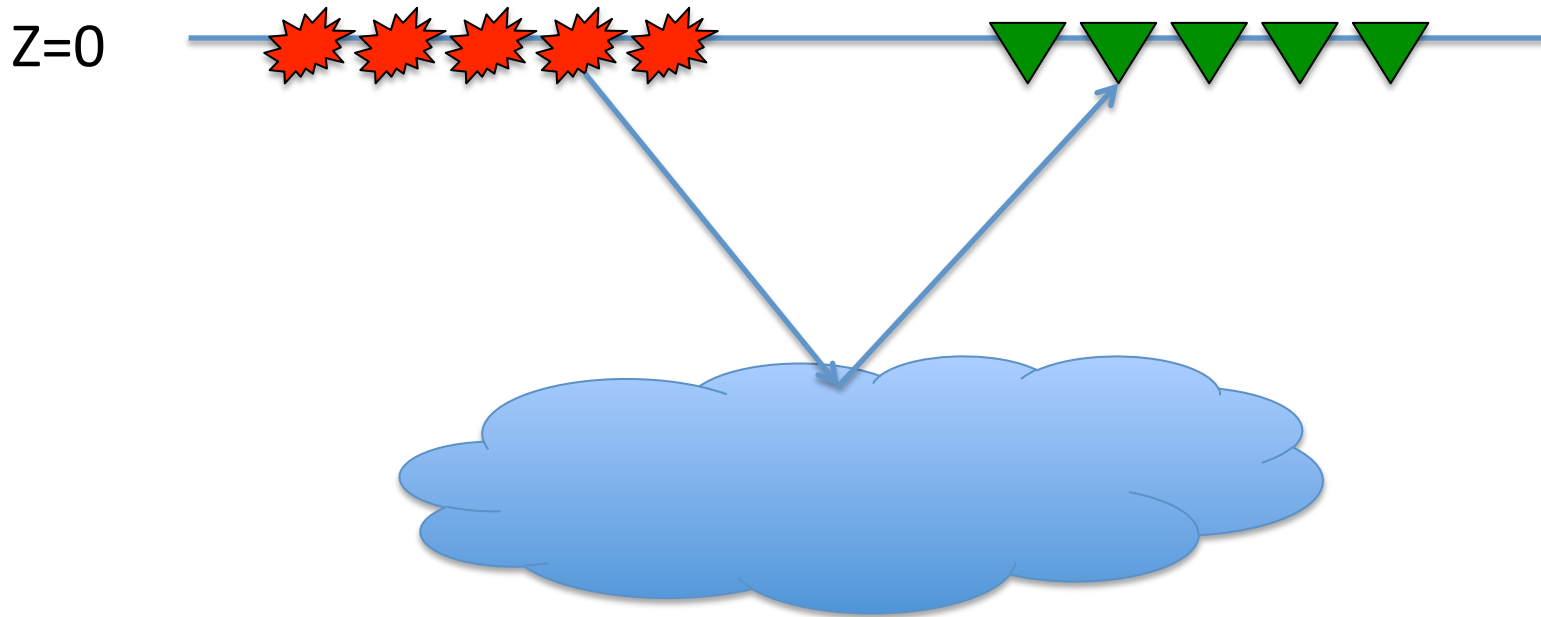
$$D(x_s, x_g, t)$$

The attenuation factor (1D earth offset data)



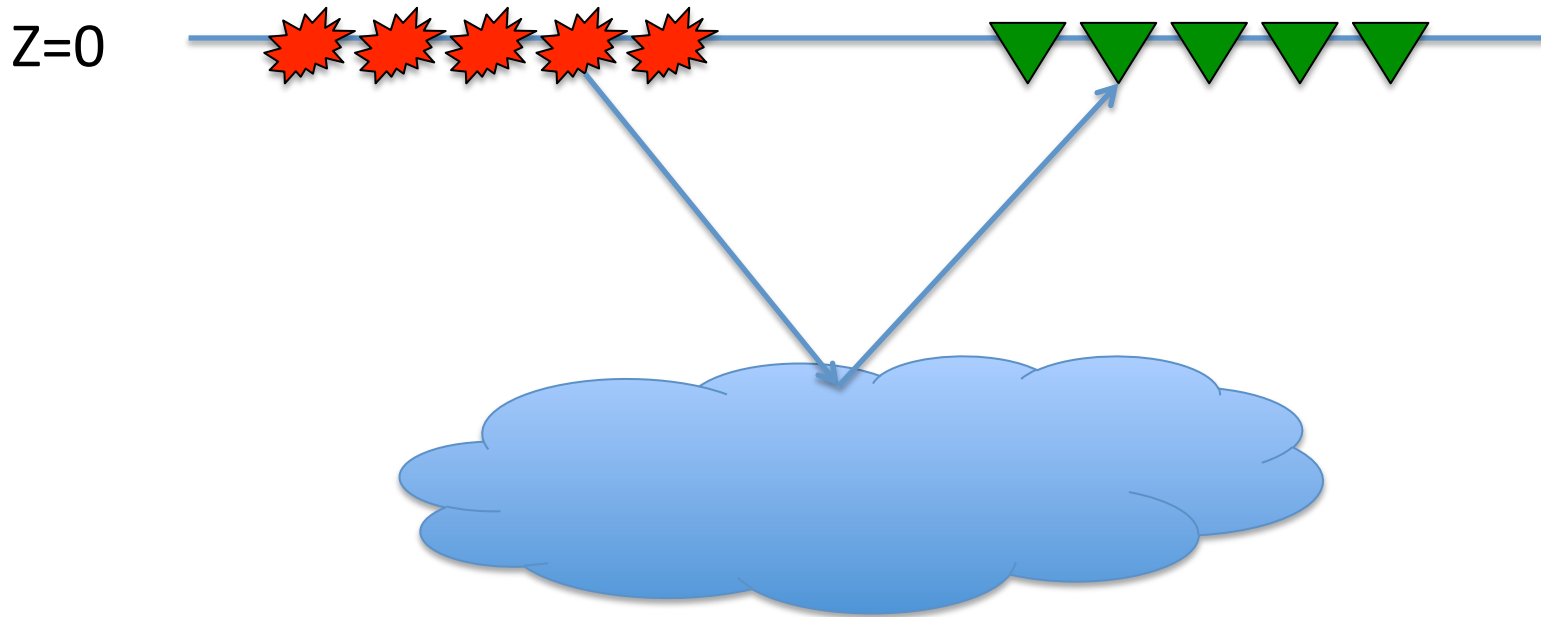
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The attenuation factor (1D earth offset data)



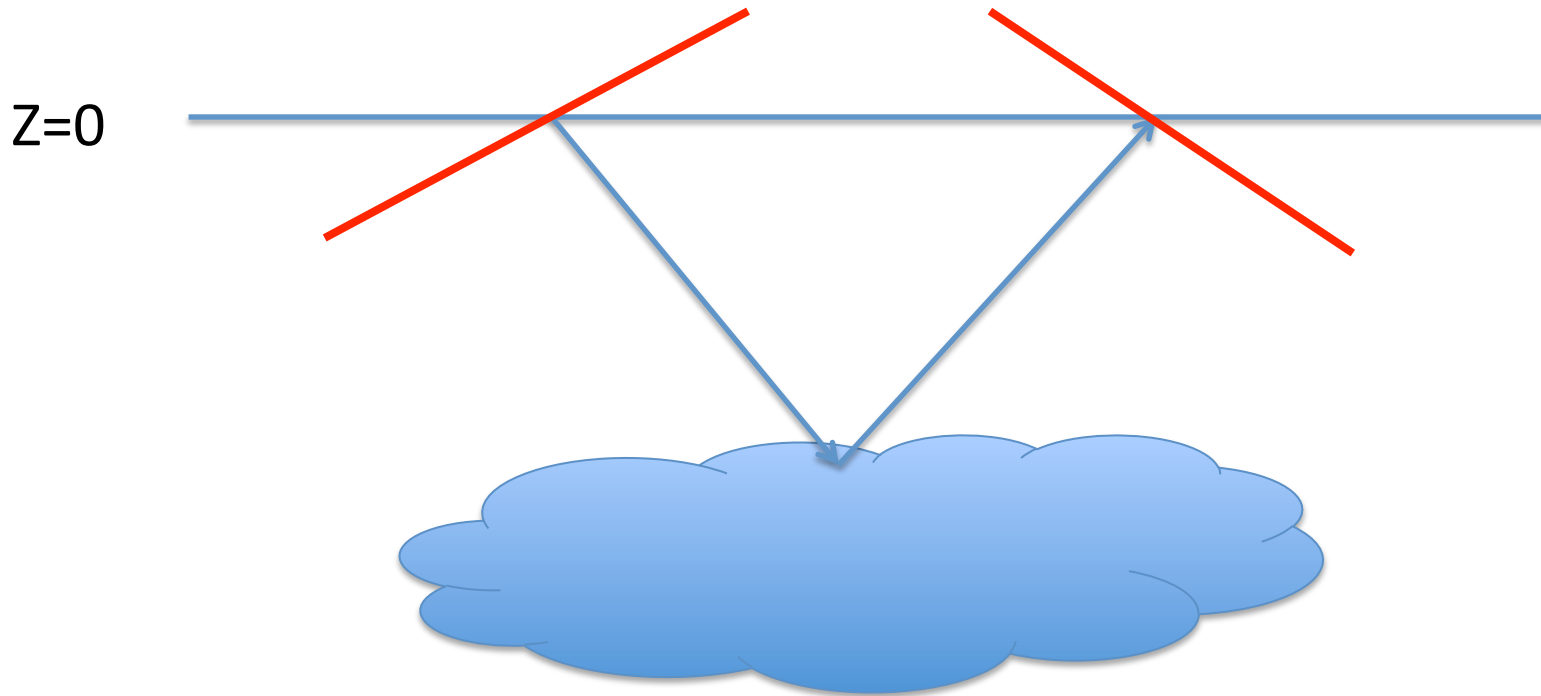
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The attenuation factor (1D earth offset data)



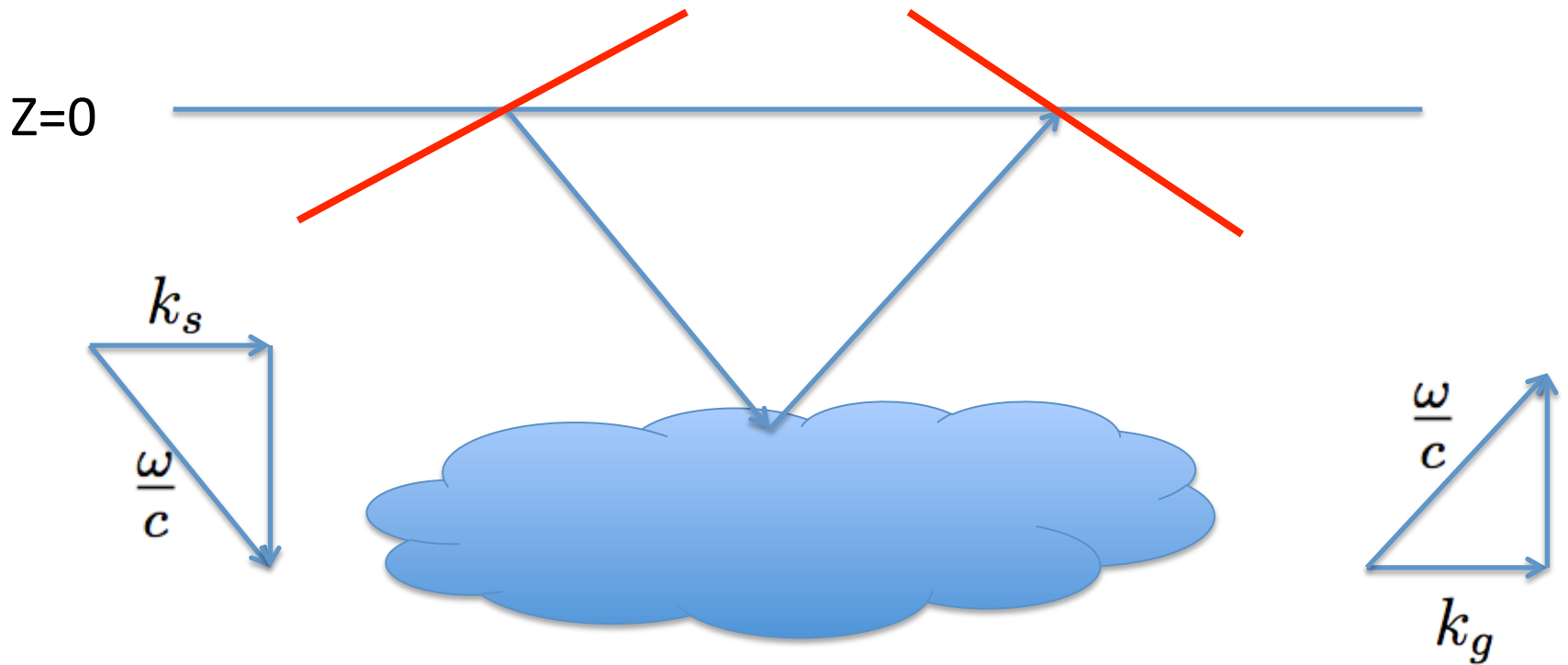
$$D(x_s, x_g, t) \longrightarrow D(k_s, k_g, \omega)$$

The attenuation factor (1D earth offset data)



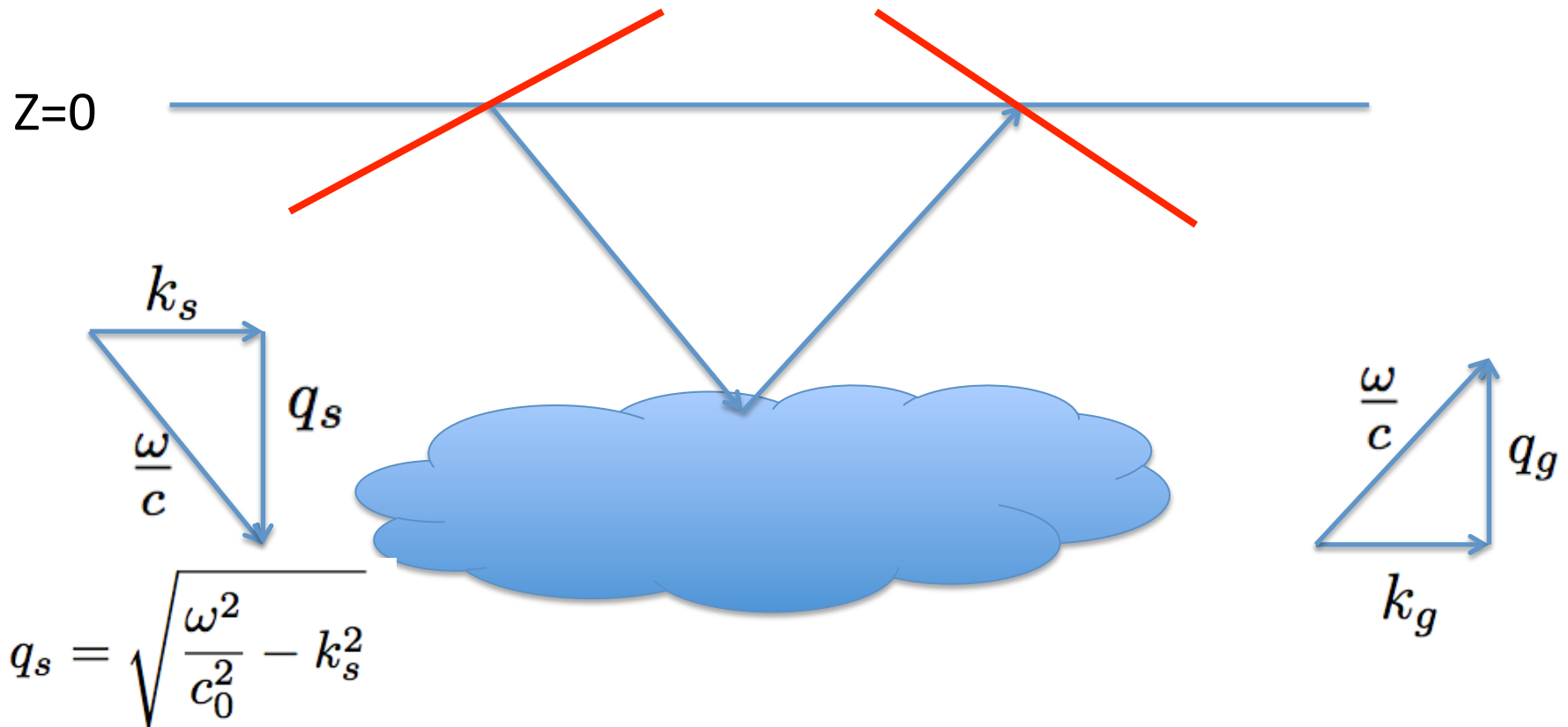
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The attenuation factor (1D earth offset data)



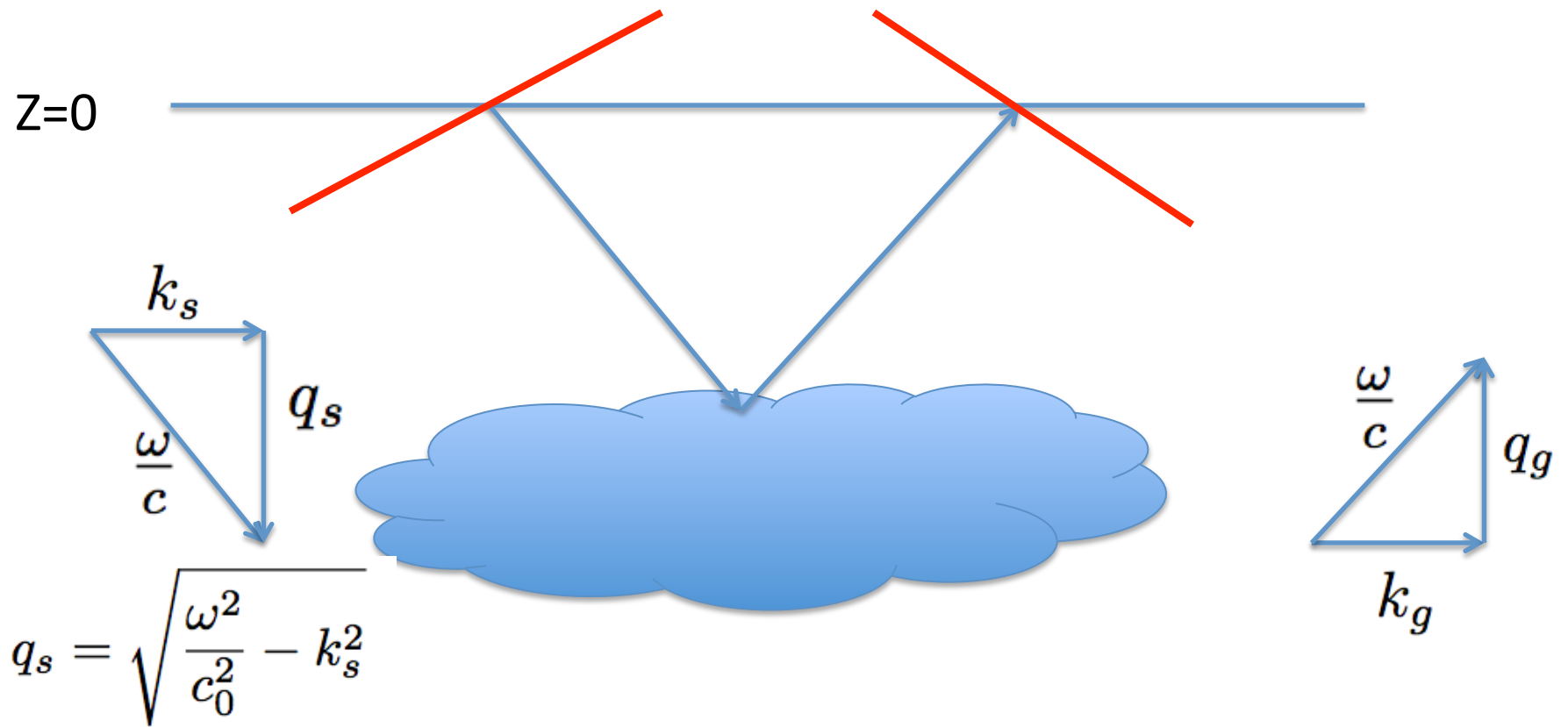
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The attenuation factor (1D earth offset data)



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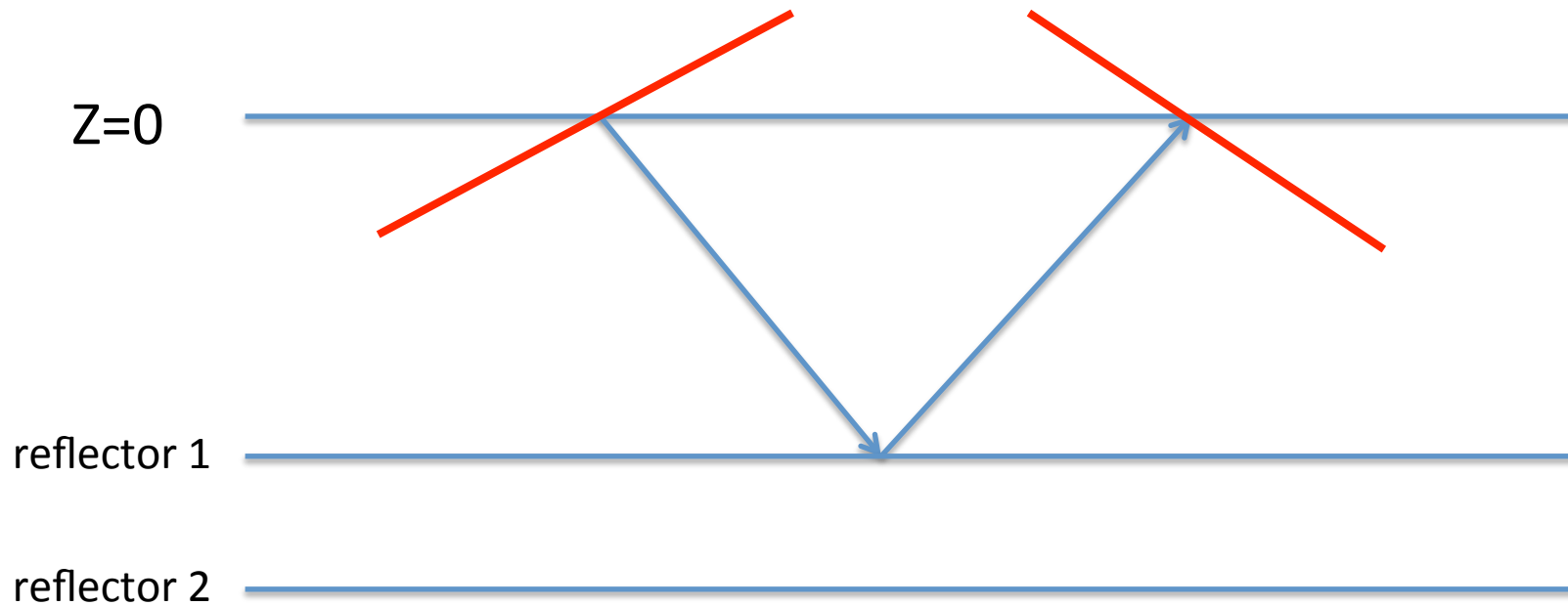
The attenuation factor (1D earth offset data)



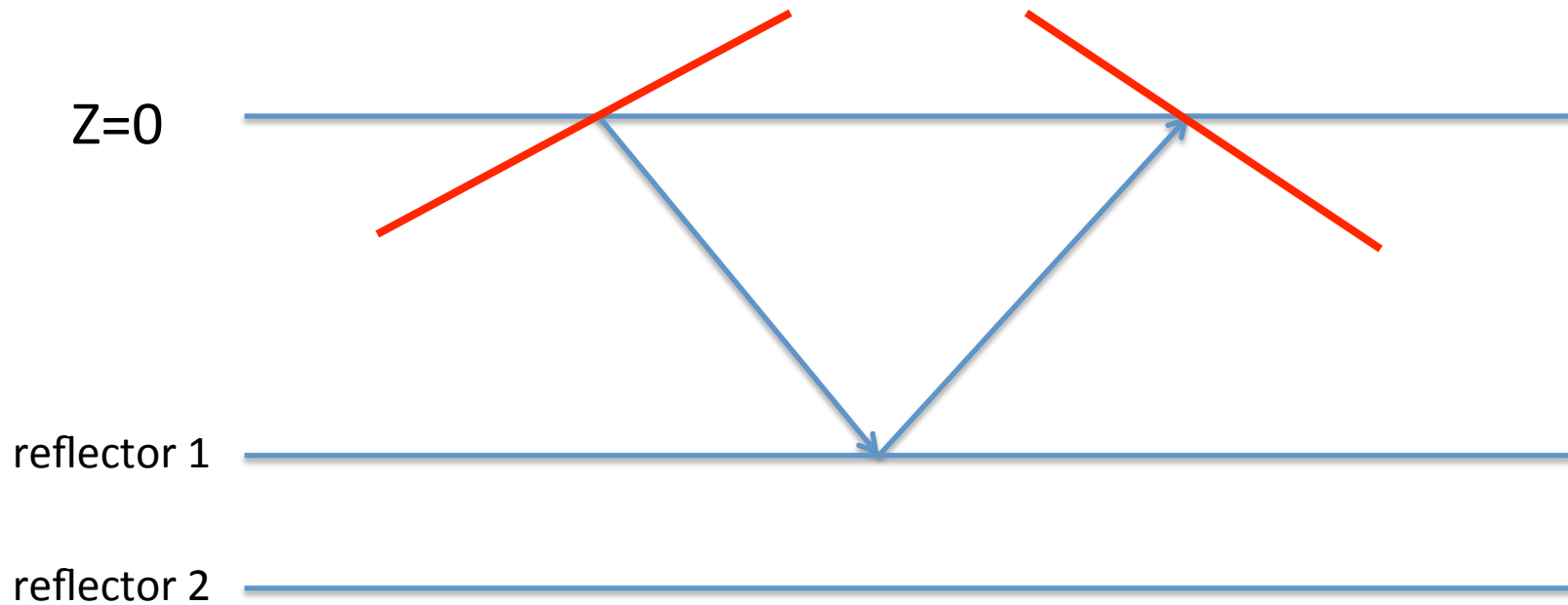
$$D(x_s, x_g, t) \longrightarrow D(k_s, k_g, \omega)$$

$$b_1(k_s, k_g, \omega) = -2iq_s D(k_s, k_g, \omega)$$

The attenuation factor (1D earth offset data)

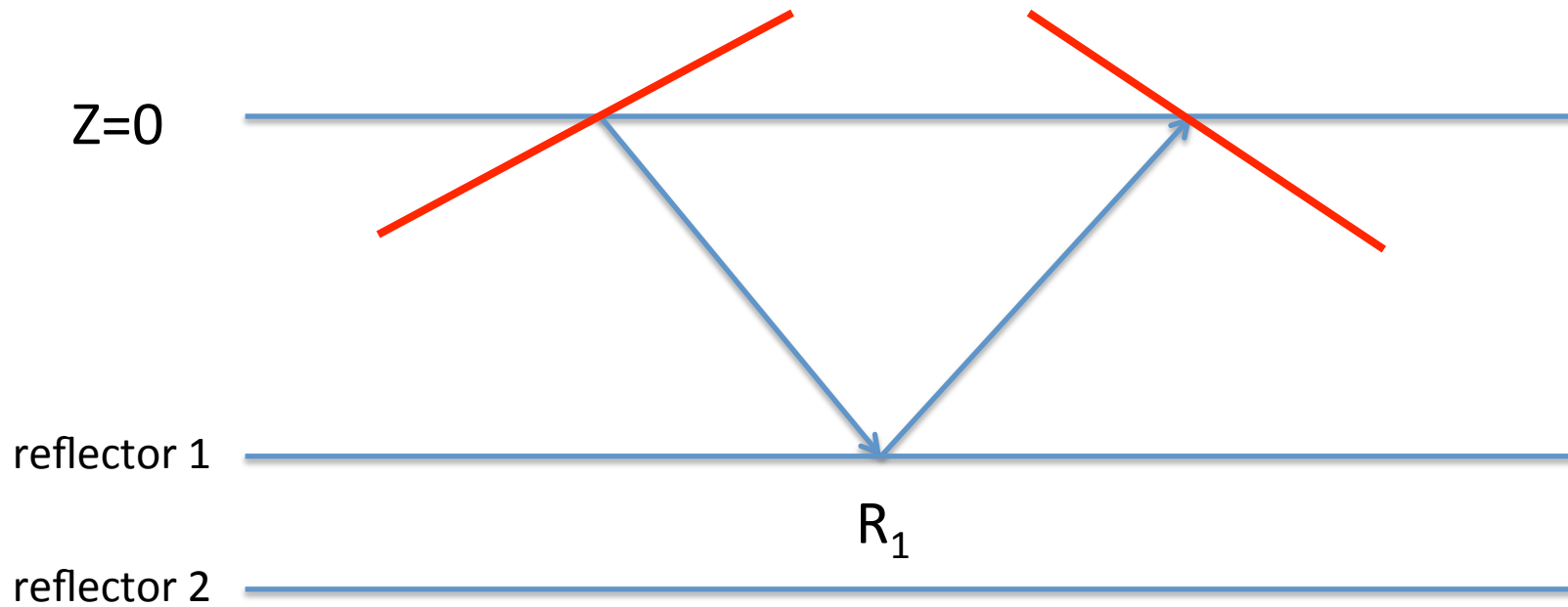


The attenuation factor (1D earth offset data)

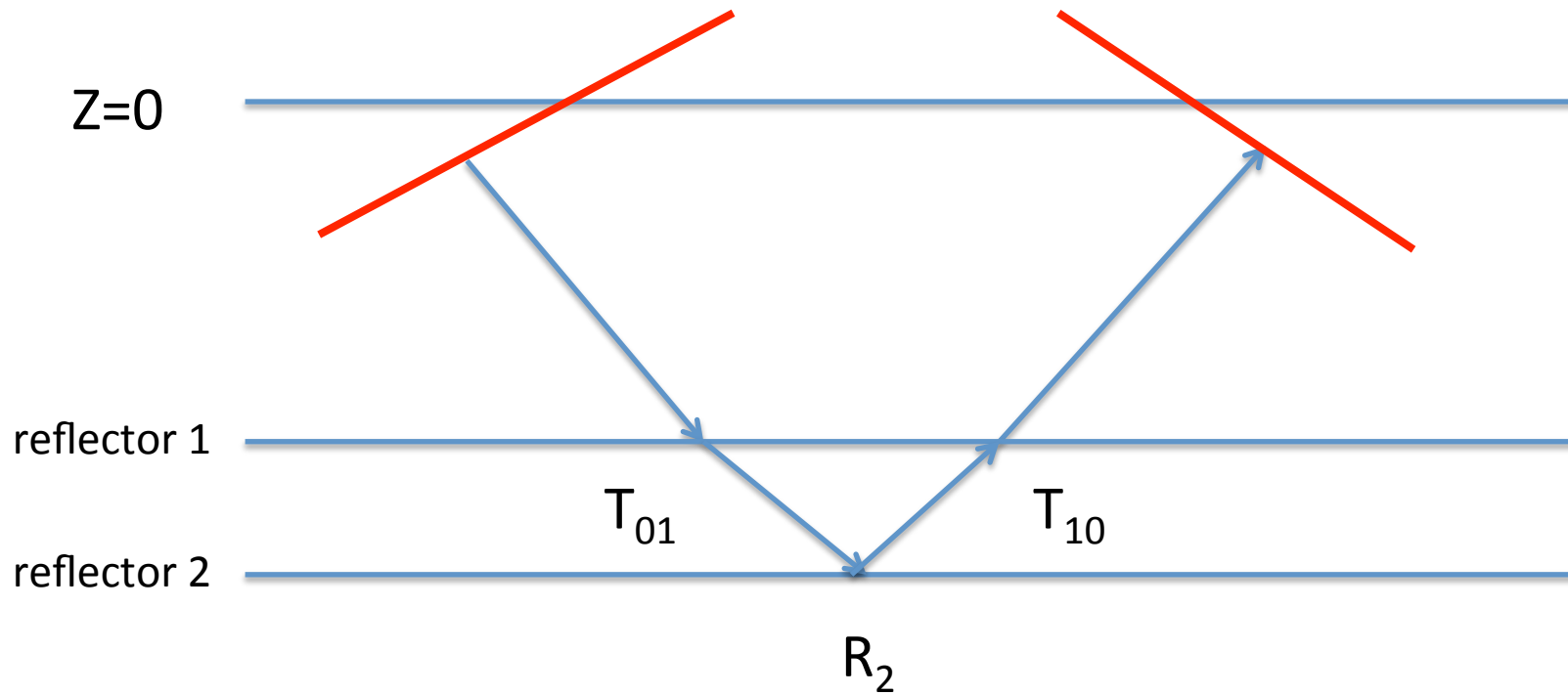


$$b_1(k_s, k_g, \omega) = -2iq_s D(k_s, k_g, \omega)$$
$$b_1(k, \omega) = -2iqD(k, \omega)$$

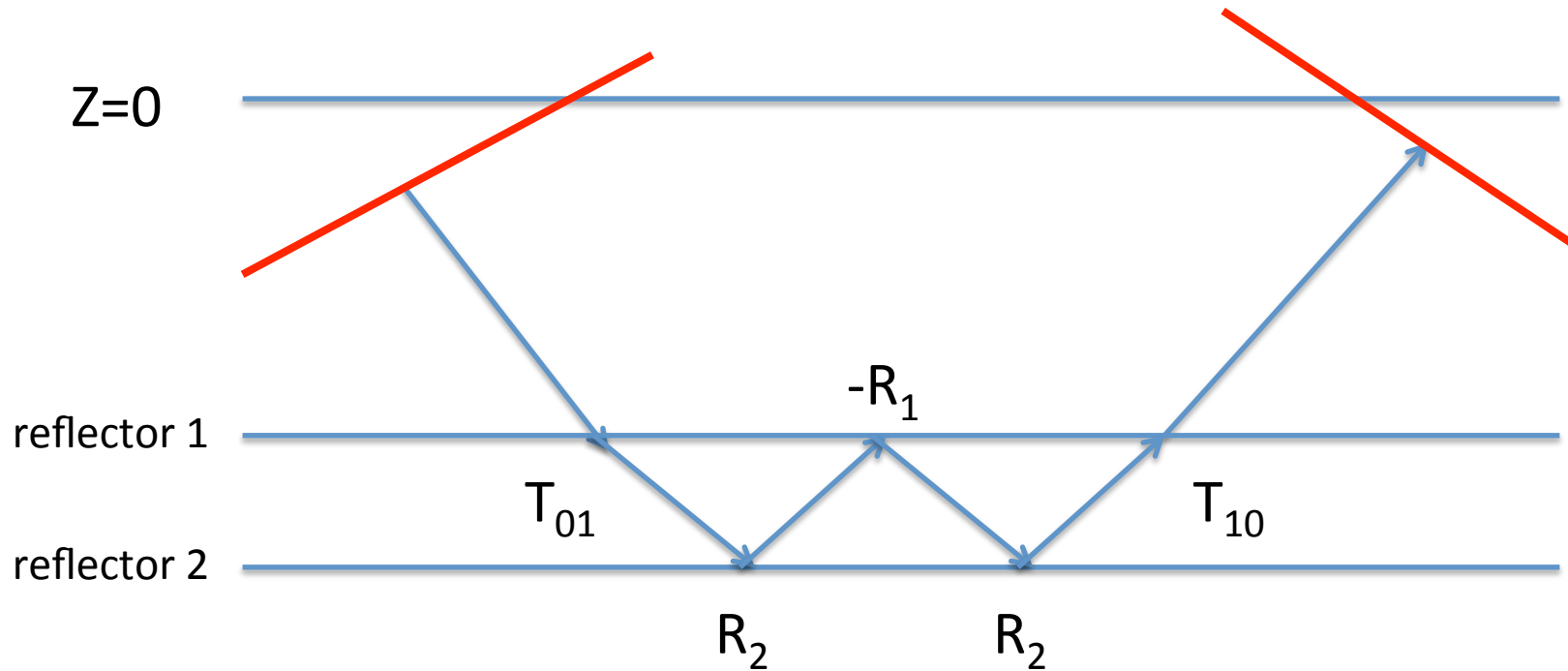
The attenuation factor (1D earth offset data)



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Each T is angle dependent and a function of incident angle.

The structure of this presentation

ISS internal multiple
attenuation algorithm

```
graph TD; A[ISS internal multiple attenuation algorithm] --> B[Attenuation Factor]; B --> C[ISS internal multiple elimination algorithm];
```

Attenuation Factor

ISS internal multiple
elimination algorithm

ISS internal multiple elimination

Internal-multiple **attenuation**
algorithm

Internal-multiple **elimination**
algorithm

The amplitude difference
between them is the
attenuation factor

ISS internal multiple elimination

Internal-multiple **attenuation**
algorithm

Internal-multiple **elimination**
algorithm

The amplitude difference
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Data b_1

ISS internal multiple elimination

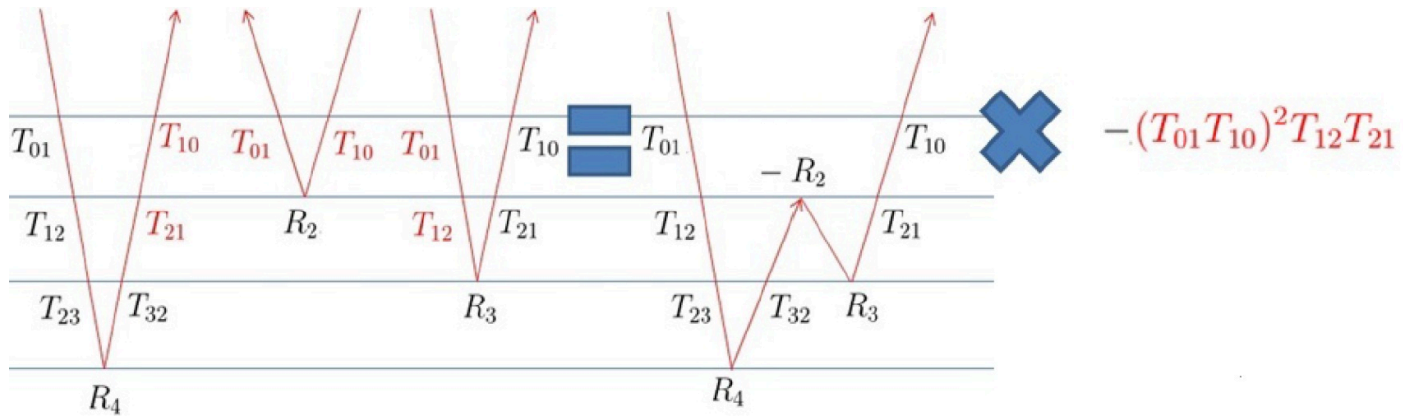
The reverse engineering way to remove first order internal multiples is to build a new function in the second integral to remove the attenuation factor.

$$b_3(k, 2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k, z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} b_1(k, z') \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k, z'')$$

ISS internal multiple elimination

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sub-events into ISS attenuator

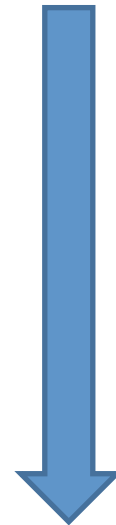
actual internal multiple

difference (AF)

ISS internal multiple elimination

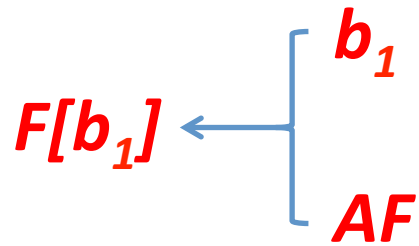
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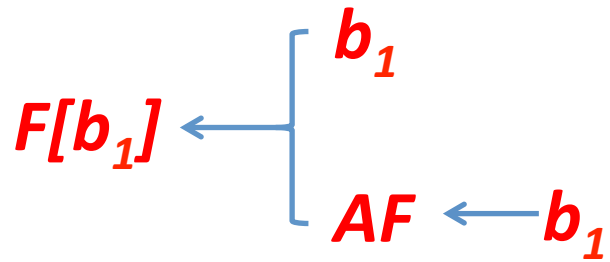


$$b_E^{IM}(k, 2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k, z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k, z')] \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k, z'')$$

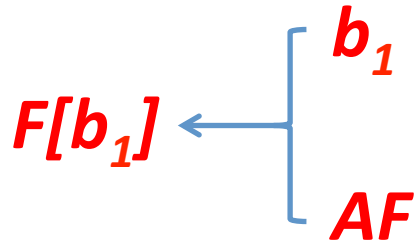
ISS internal multiple elimination



ISS internal multiple elimination



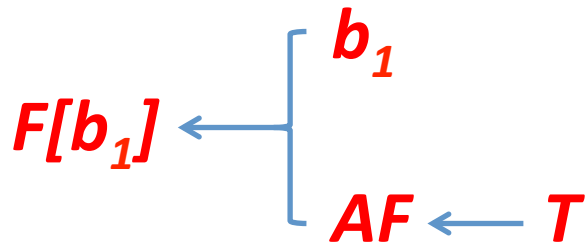
ISS internal multiple elimination



The amplitude of events in $F[b_1]$ should be:

$$\frac{R'_i}{AF_i}$$

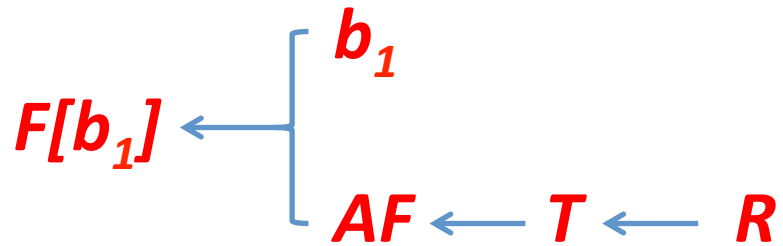
ISS internal multiple elimination



The amplitude of events in $F[b_1]$ should be:

$$\frac{R'_i}{AF_i} = \frac{R'_i}{(T_{01}T_{10})^2(T_{12}T_{21})^2 \cdots (T_{i-2,i-1}T_{i-1,i-2})^2(T_{i-1,i}T_{i,i-1})}$$

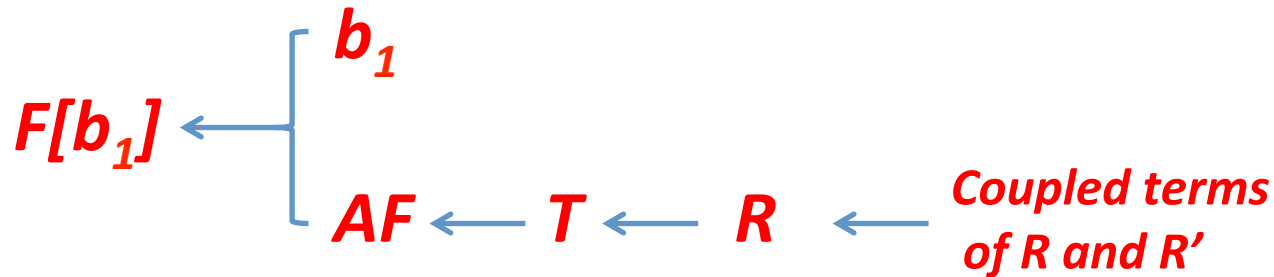
ISS internal multiple elimination



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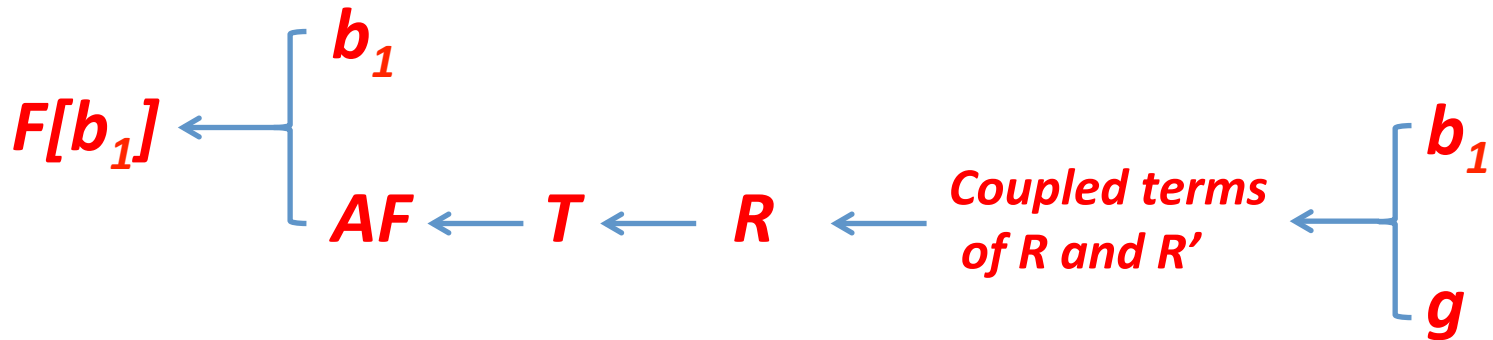
ISS internal multiple elimination



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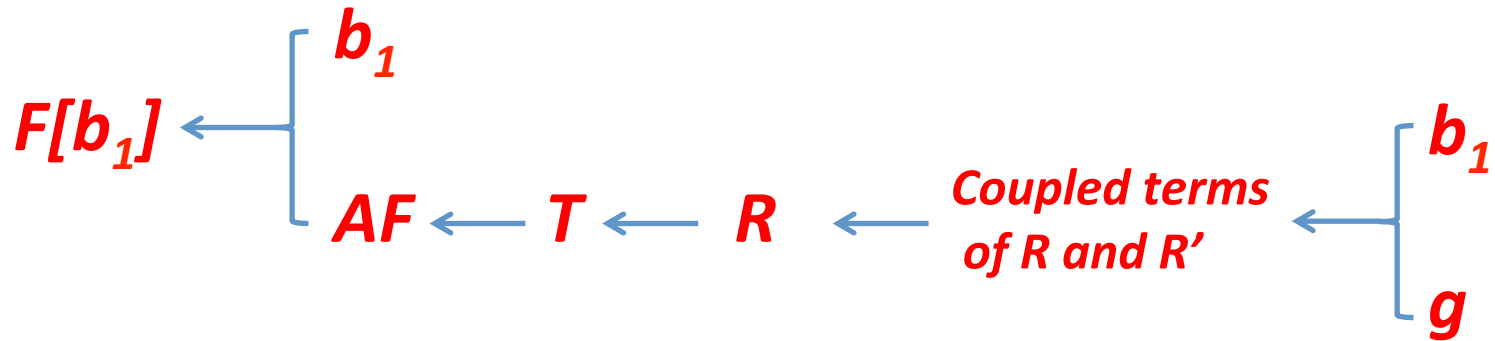
ISS internal multiple elimination



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ISS internal multiple elimination

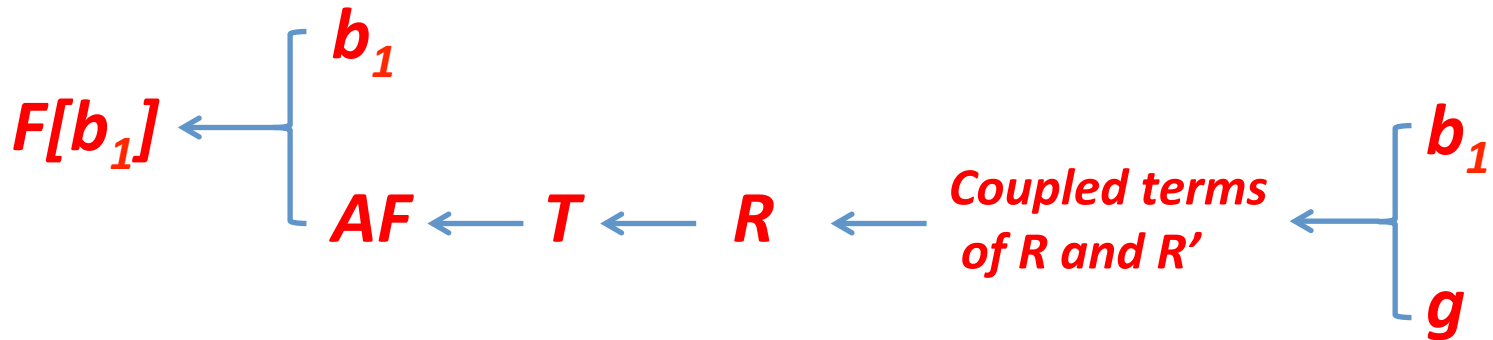


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 \end{aligned}$$

$$\int_{z'-\epsilon}^{z'+\epsilon} dz'' g(k, z'') e^{iq'z''} \int_{z''-\epsilon}^{z''+\epsilon} dz''' g^*(k, z''') e^{-iq'z'''} \longrightarrow R_i^2$$

ISS internal multiple elimination



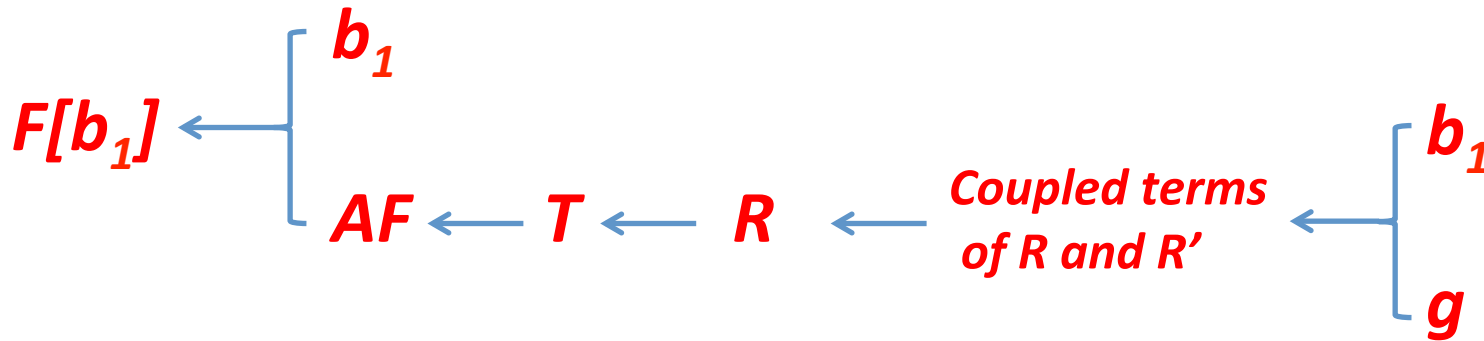
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This term is an extension of the self-interaction term in Herrera(2012)

ISS internal multiple elimination



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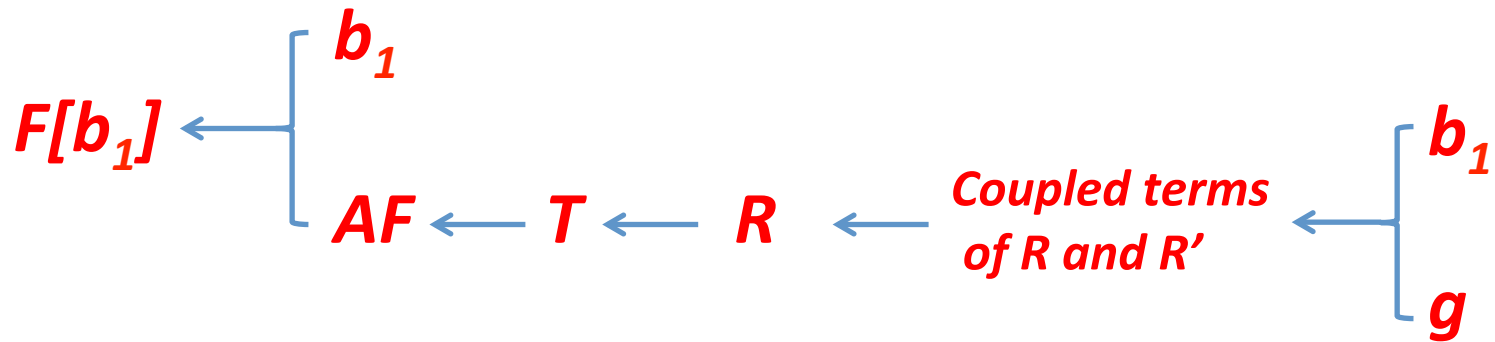
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This term is an extension of the self-interaction term in Herrera(2012)

$$\int_{-\infty}^{z-\epsilon} dz' b_1(k, z') e^{iq'z'} \int_{z'-\epsilon}^{z'+\epsilon} dz'' g^*(k, z'') e^{-iq'z''} \longrightarrow R_1R_1 + R_2R_2 + \cdots + R_{i-1}R_{i-1}$$

ISS internal multiple elimination

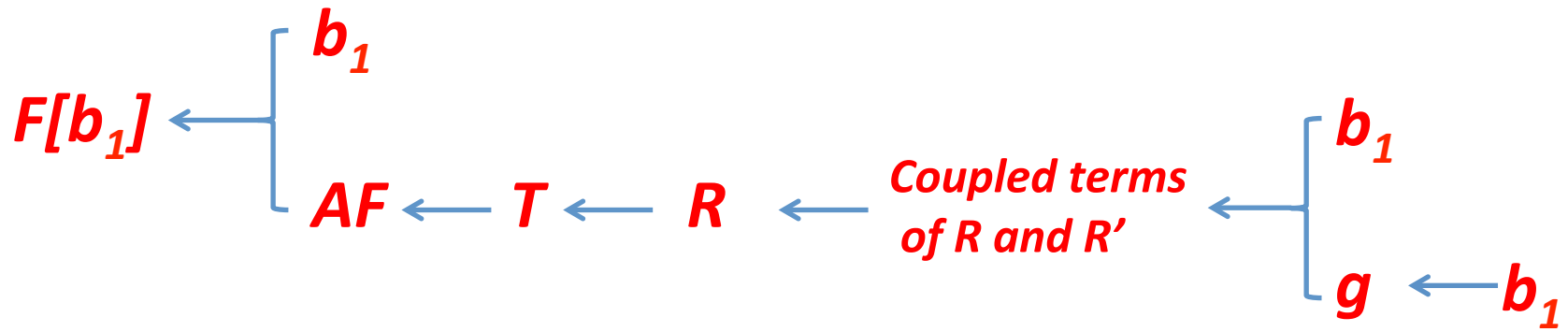


$$F[b_1(k, z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times$$

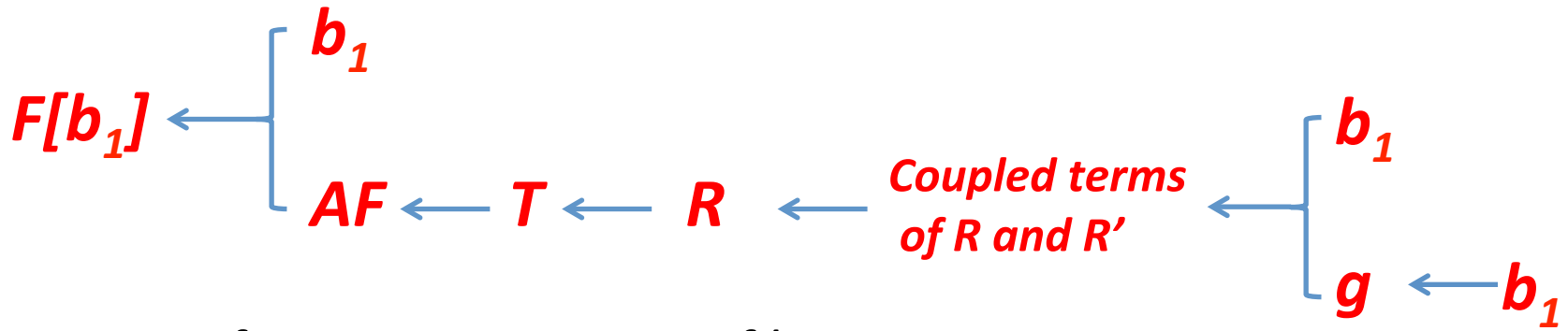
$$e^{-iq'z} e^{iq'z'} b_1(k, z')$$

$$[1 - \int_{-\infty}^{z'-\epsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\epsilon}^{z''+\epsilon} dz''' g^*(k, z''') e^{-iq'z'''}] [1 - \int_{z'-\epsilon}^{z'+\epsilon} dz'' g(k, z'') e^{iq'z''} \int_{z''-\epsilon}^{z''+\epsilon} dz''' g^*(k, z''') e^{-iq'z'''}]$$

ISS internal multiple elimination

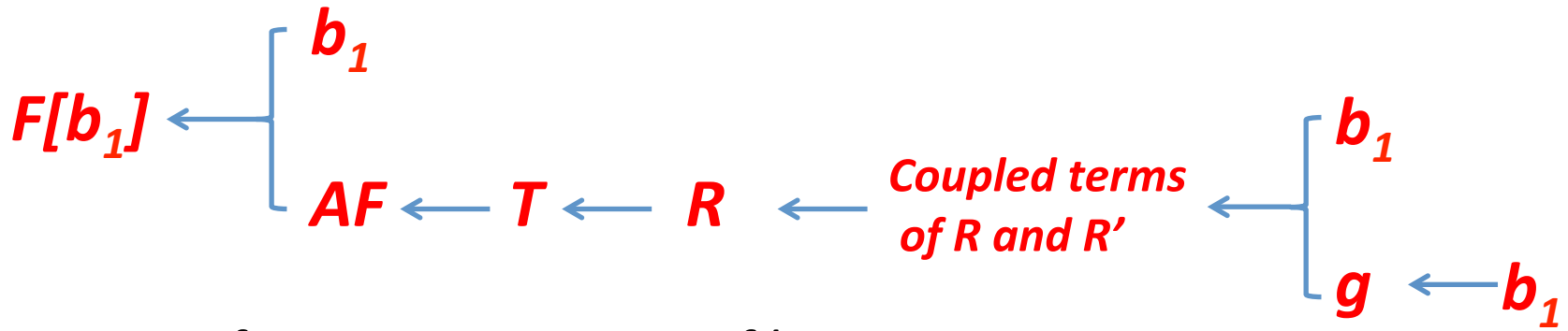


ISS internal multiple elimination



Construct function g in terms of b_1 :

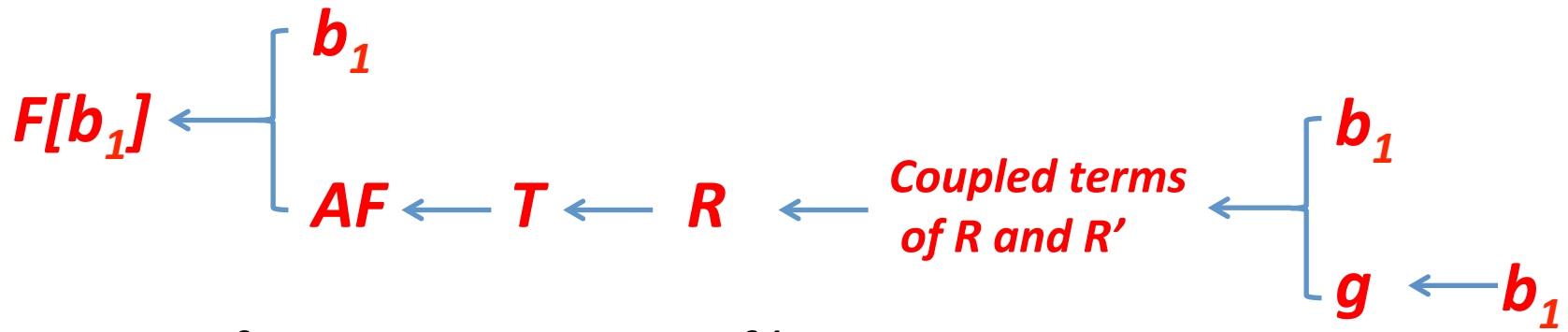
ISS internal multiple elimination



Construct function g in terms of b_1 :

R_i

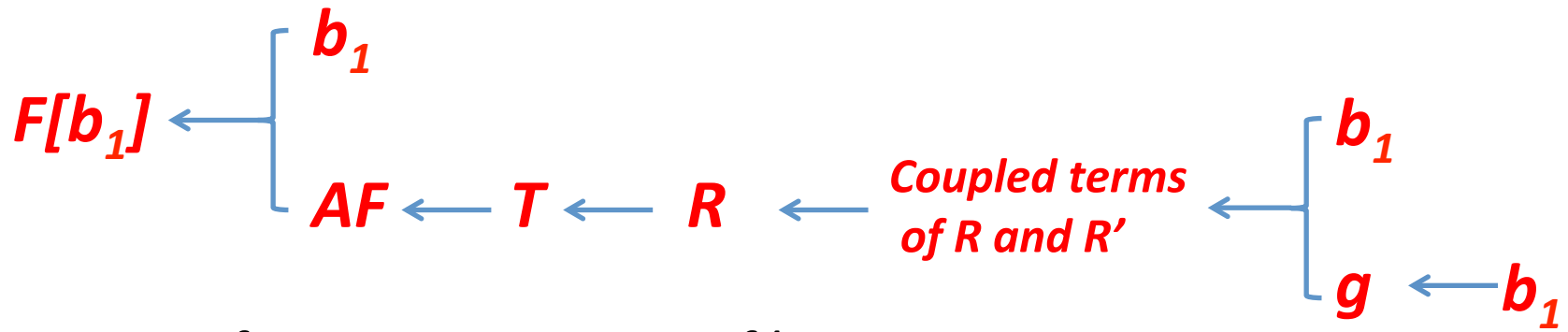
ISS internal multiple elimination



Construct function g in terms of b_1 :

$$\begin{aligned}
 R_i &= \frac{R'_i}{(1 - R_1^2)(1 - R_2^2) \cdots (1 - R_{i-2}^2)(1 - R_{i-1}^2)} \\
 &= \frac{R'_i}{1 - R_1 R_1 - R'_2 R_2 - \cdots - R'_{i-1} R_{i-1}}
 \end{aligned}$$

ISS internal multiple elimination



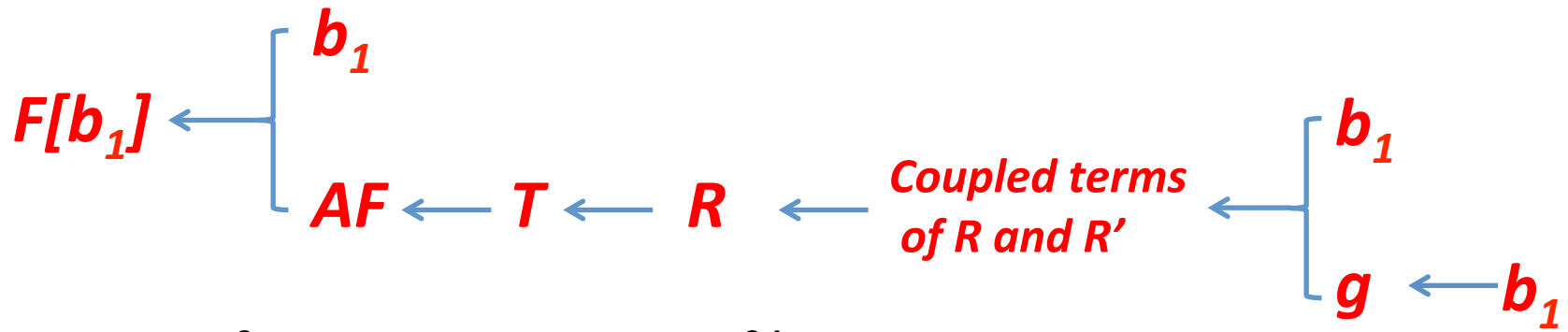
Construct function g in terms of b_1 :

$$R_i = \frac{R'_i}{(1 - R_1^2)(1 - R_2^2) \cdots (1 - R_{i-2}^2)(1 - R_{i-1}^2)}$$

$$= \frac{R'_i}{1 - R_1 R_1 - R'_2 R_2 - \cdots - R'_{i-1} R_{i-1}}$$

$$\int_{-\infty}^{z-\varepsilon} dz' b_1(k, z') e^{iq'z'} \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g^*(k, z'') e^{-iq'z''} \longrightarrow R_1 R_1 + R'_2 R_2 + \cdots + R'_{i-1} R_{i-1}$$

ISS internal multiple elimination



Construct function g in terms of b_1 :

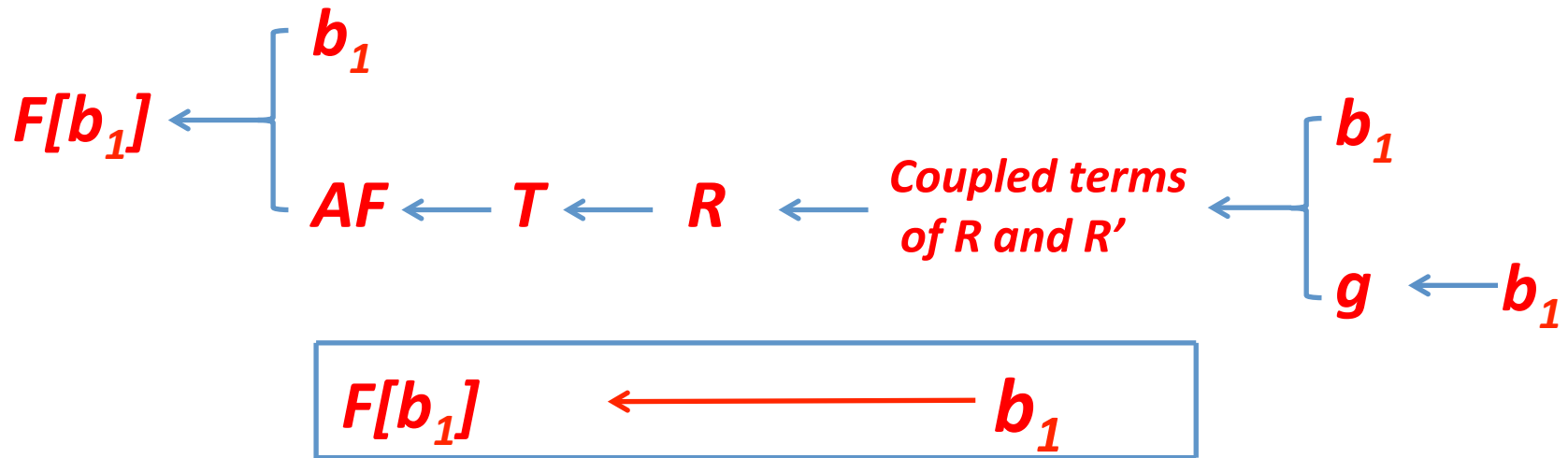
$$R_i = \frac{R'_i}{(1 - R_1^2)(1 - R_2^2) \cdots (1 - R_{i-2}^2)(1 - R_{i-1}^2)}$$

$$= \frac{R'_i}{1 - R_1 R_1 - R_2 R_2 - \cdots - R'_{i-1} R_{i-1}}$$

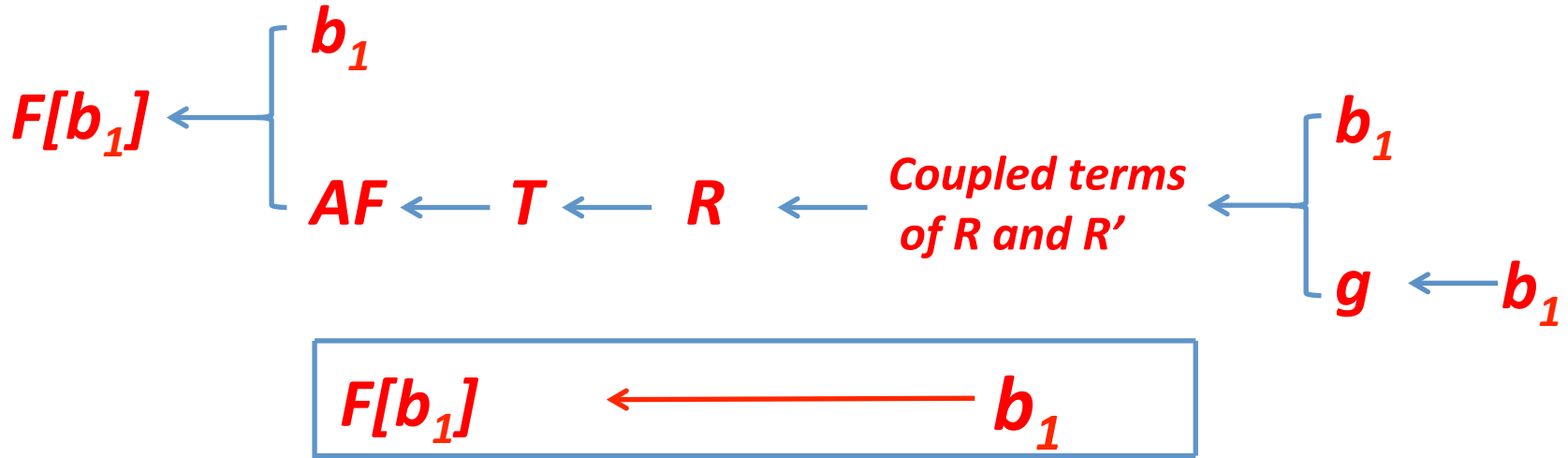
$$\int_{-\infty}^{z-\epsilon} dz' b_1(k, z') e^{iq' z'} \int_{z'-\epsilon}^{z'+\epsilon} dz'' g^*(k, z'') e^{-iq' z''} \longrightarrow R_1 R_1 + R'_2 R_2 + \cdots + R'_{i-1} R_{i-1}$$

$$g(k, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \frac{e^{-iq' z} e^{iq' z'} b_1(k, z')}{1 - \int_{-\infty}^{z'-\epsilon} dz'' b_1(k, z'') e^{iq' z''} \int_{z''-\epsilon}^{z''+\epsilon} dz''' g^*(k, z''') e^{-iq' z'''}}$$

ISS internal multiple elimination



ISS internal multiple elimination



$$b_E^{IM}(k, 2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k, z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k, z')] \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k, z'')$$

$$F[b_1(k, z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]^2 [1 - \int_{z'+\varepsilon}^{z'+\varepsilon} dz'' g(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]}$$

$$g(k, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}}$$

ISS internal multiple elimination

Flow chart of the elimination algorithm

Input: data $D(k, \omega)$

$$b_1(k, \omega) = -2iqD(k, \omega)$$

$$g(k, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}}$$

$$F[b_1(k, z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]^2 [1 - \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]}$$

$$b_E^{IM}(k, 2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k, z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k, z')] \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k, z'')$$

Output: internal multiples with correct time and amplitude $b_E^{IM}(k, 2q)$

ISS internal multiple elimination

Test 1

Test 1: 1D normal incidence

ISS internal multiple elimination

Test 1 — model

$$V=1500\text{m/s} \quad \rho=1.0\text{g/cm}^3$$

500m

$$V=1700\text{m/s} \quad \rho=1.8\text{g/cm}^3$$

1700m

$$V=1700\text{m/s} \quad \rho=1.0\text{g/cm}^3$$

2700m

$$V=3500\text{m/s} \quad \rho=4.0\text{g/cm}^3$$

5700m

$$V=5000\text{m/s} \quad \rho=4.0\text{g/cm}^3$$

ISS internal multiple elimination

Test 1

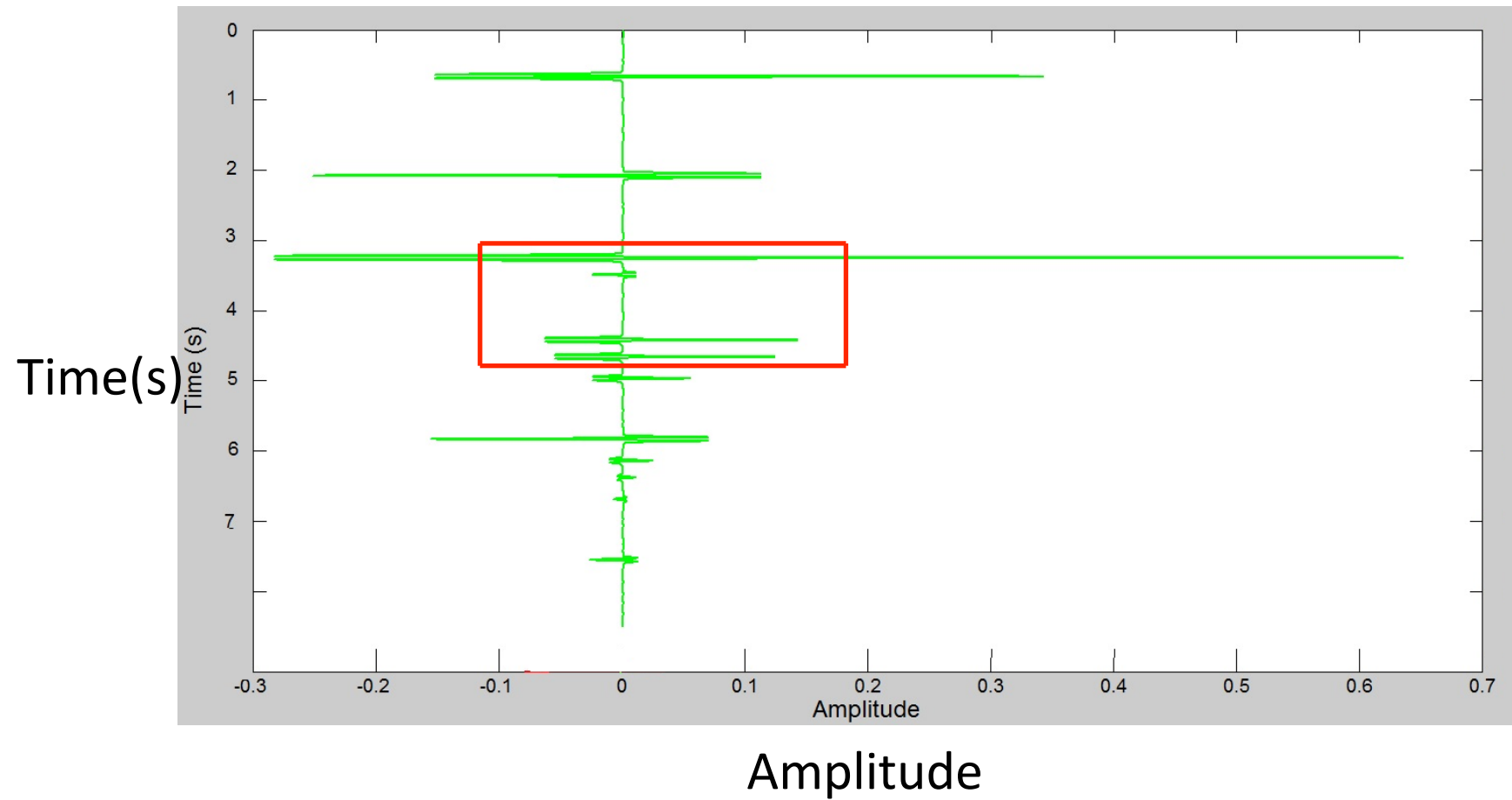
- A. Test for perfect data.
- B. Test for bandlimited data.
- C. Test for data with white noise.

ISS internal multiple elimination

Test 1—perfect data

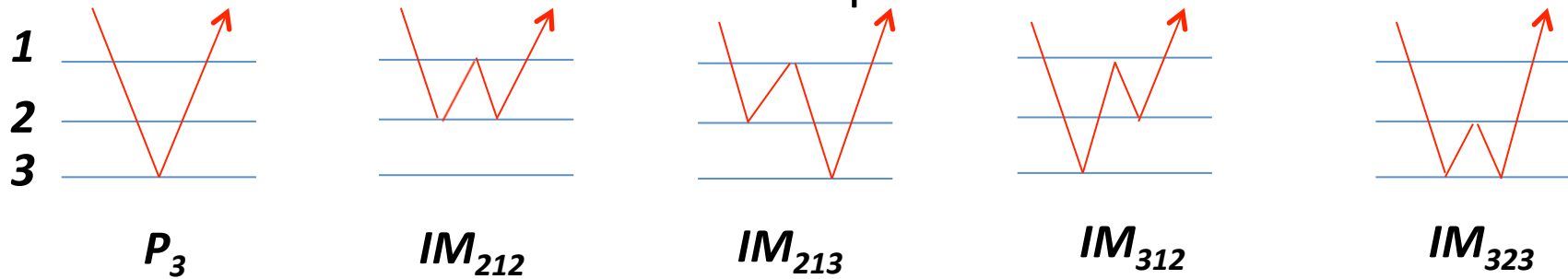
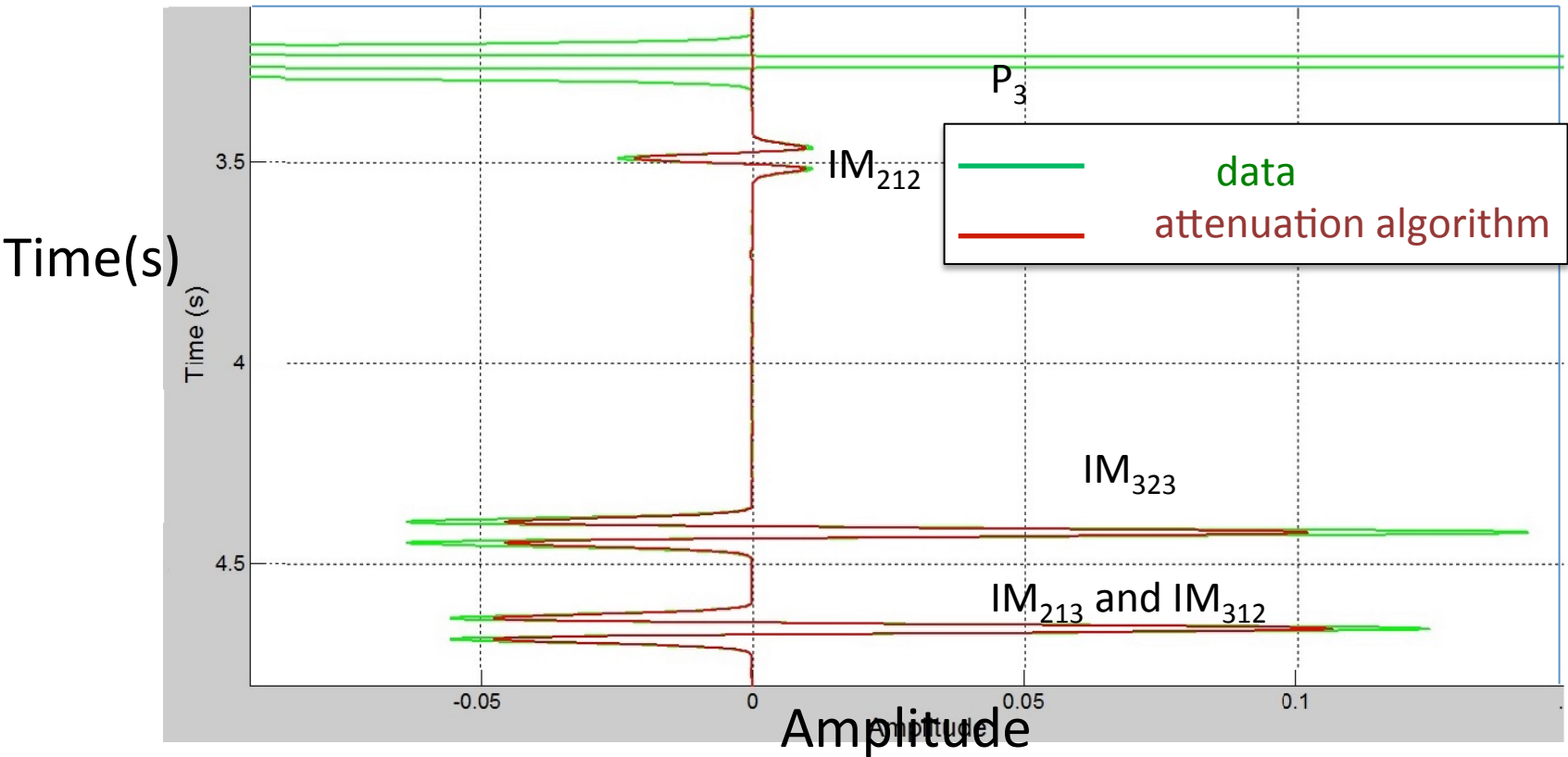
- A. Test for perfect data.
- B. Test for bandlimited data.
- C. Test for data with white noise.

ISS internal multiple elimination
Test 1—input data (1D normal incidence)



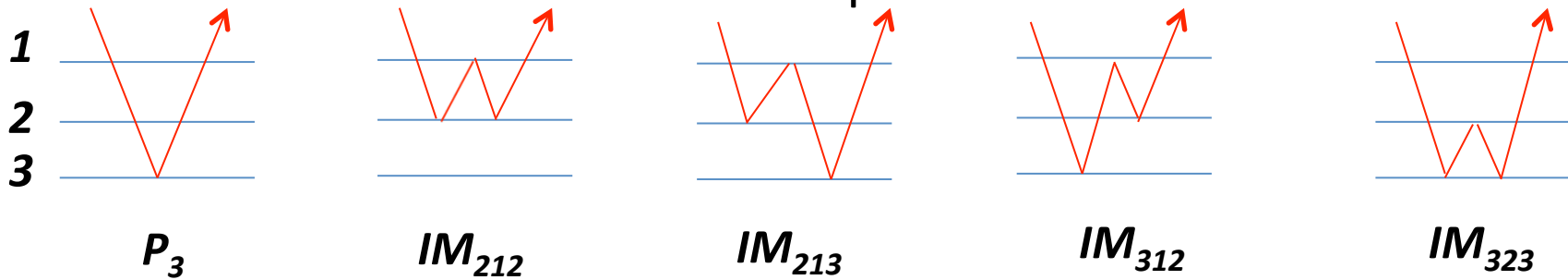
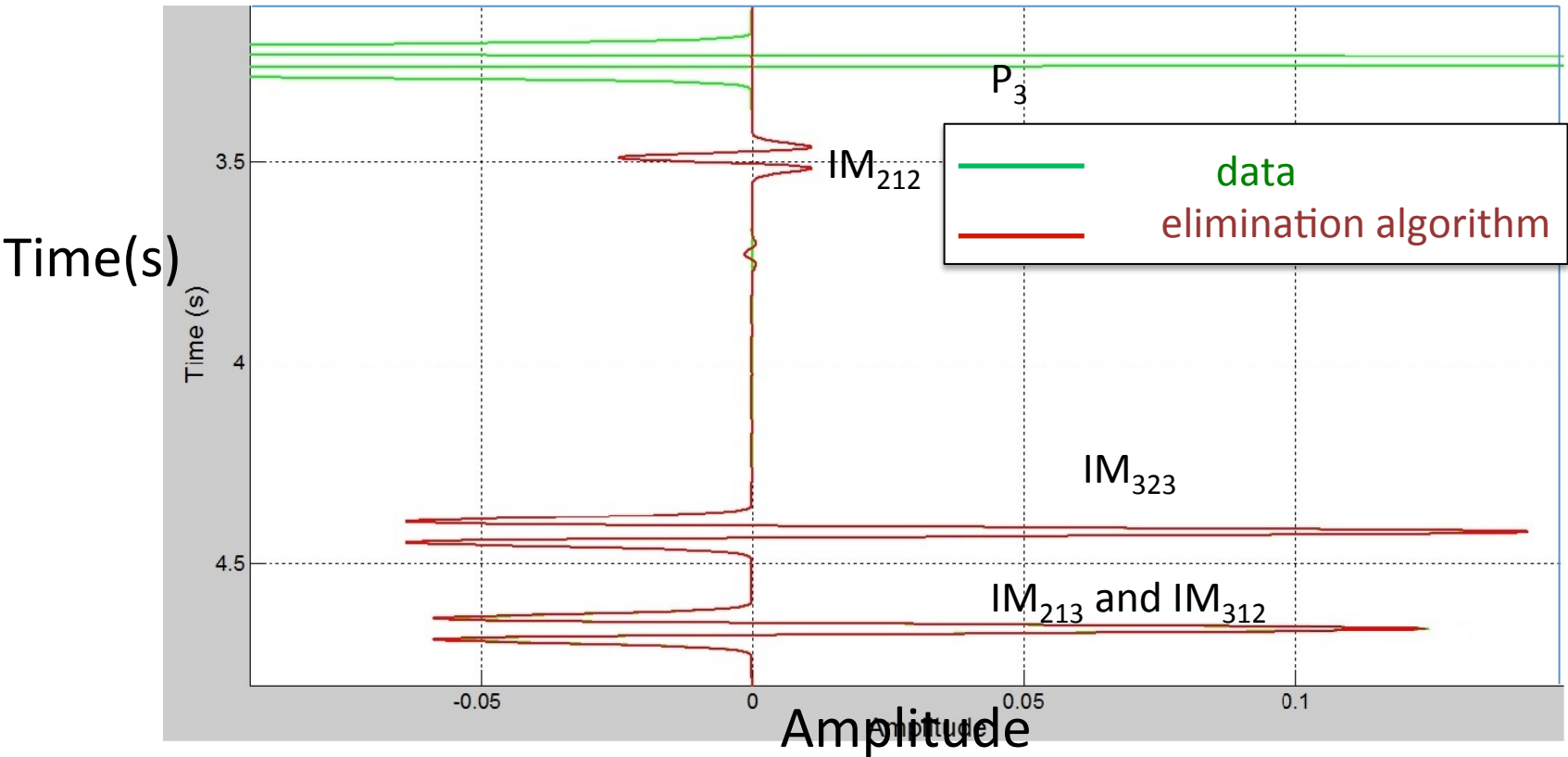
ISS internal multiple elimination

Test 1 — perfect data, attenuation algorithm prediction



ISS internal multiple elimination

Test 1 — perfect data, elimination algorithm prediction



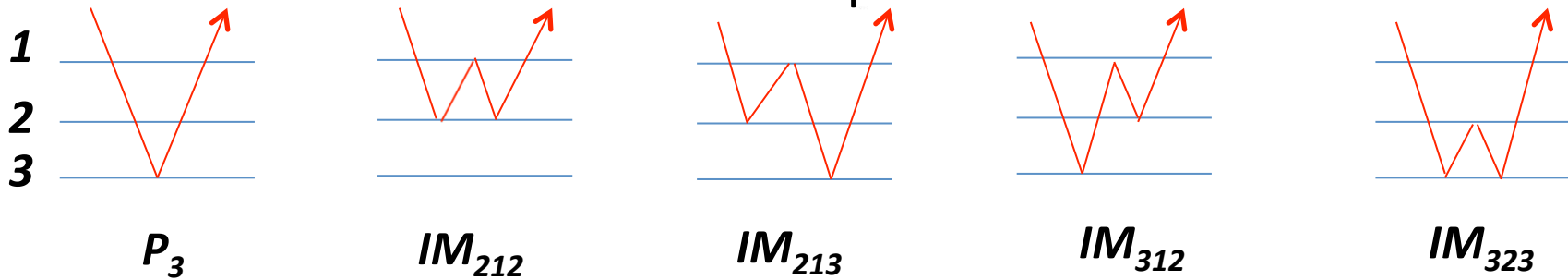
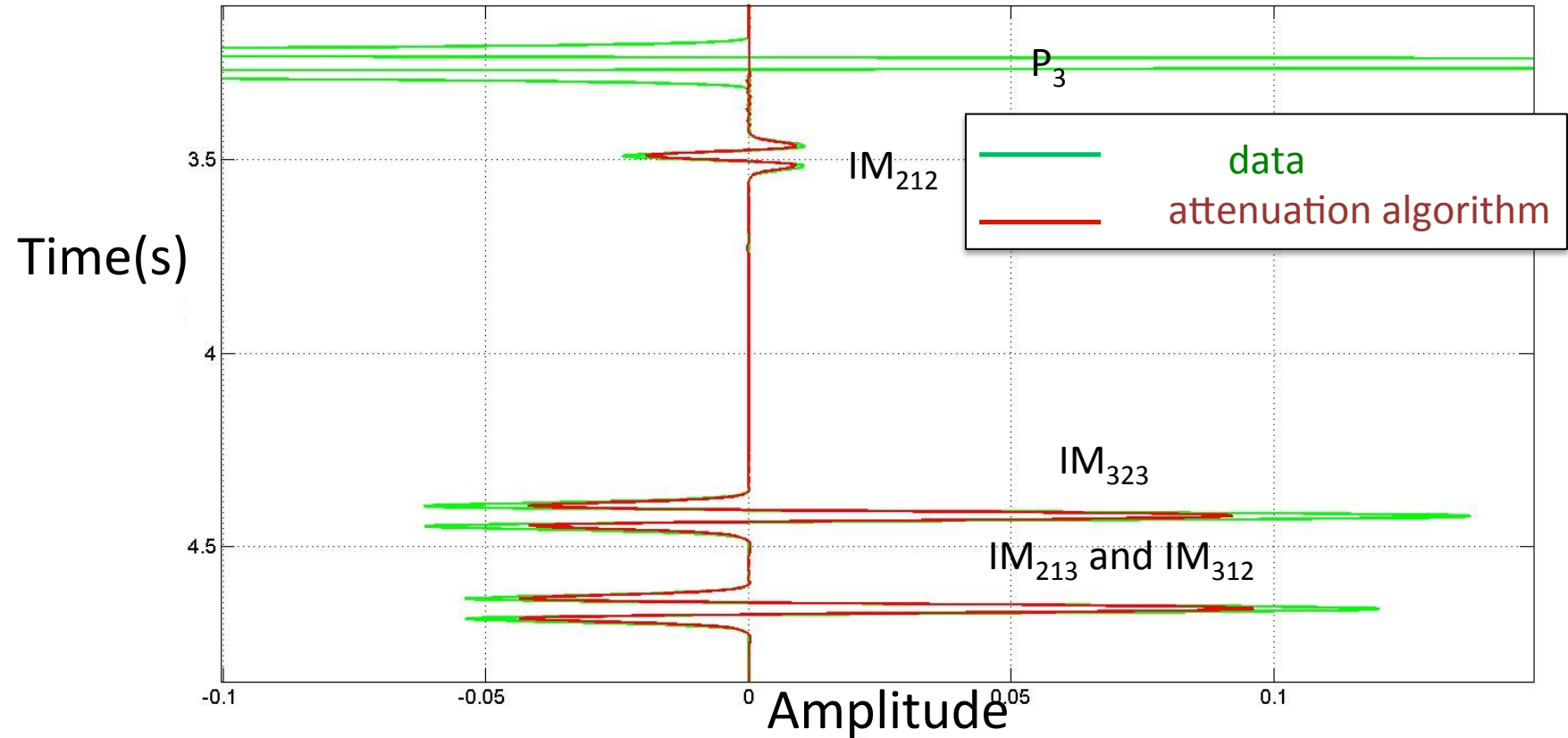
ISS internal multiple elimination

Test 1—bandlimited data

- A. Test for perfect data.
- B. Test for bandlimited data.**
- C. Test for data with white noise.

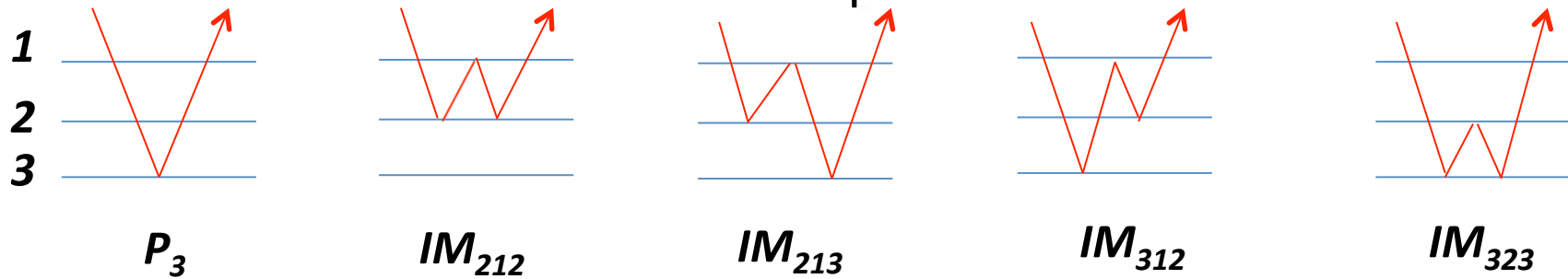
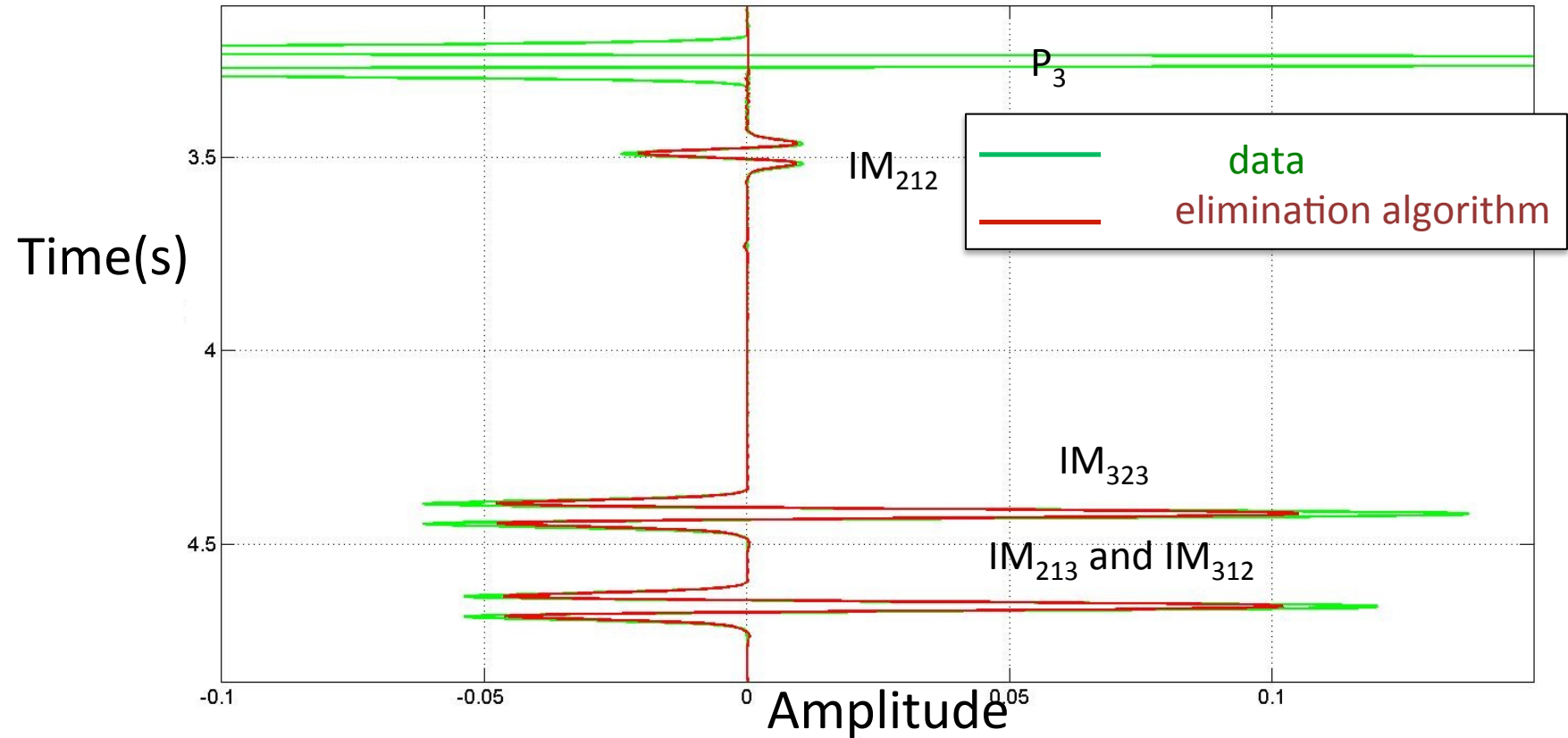
ISS internal multiple elimination

Test 1 — perfect data, attenuation algorithm prediction



ISS internal multiple elimination

Test 1 — perfect data, elimination algorithm prediction



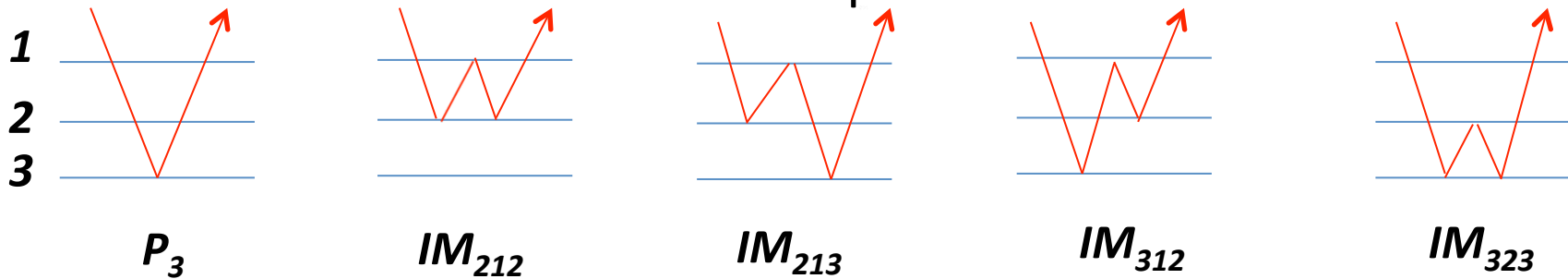
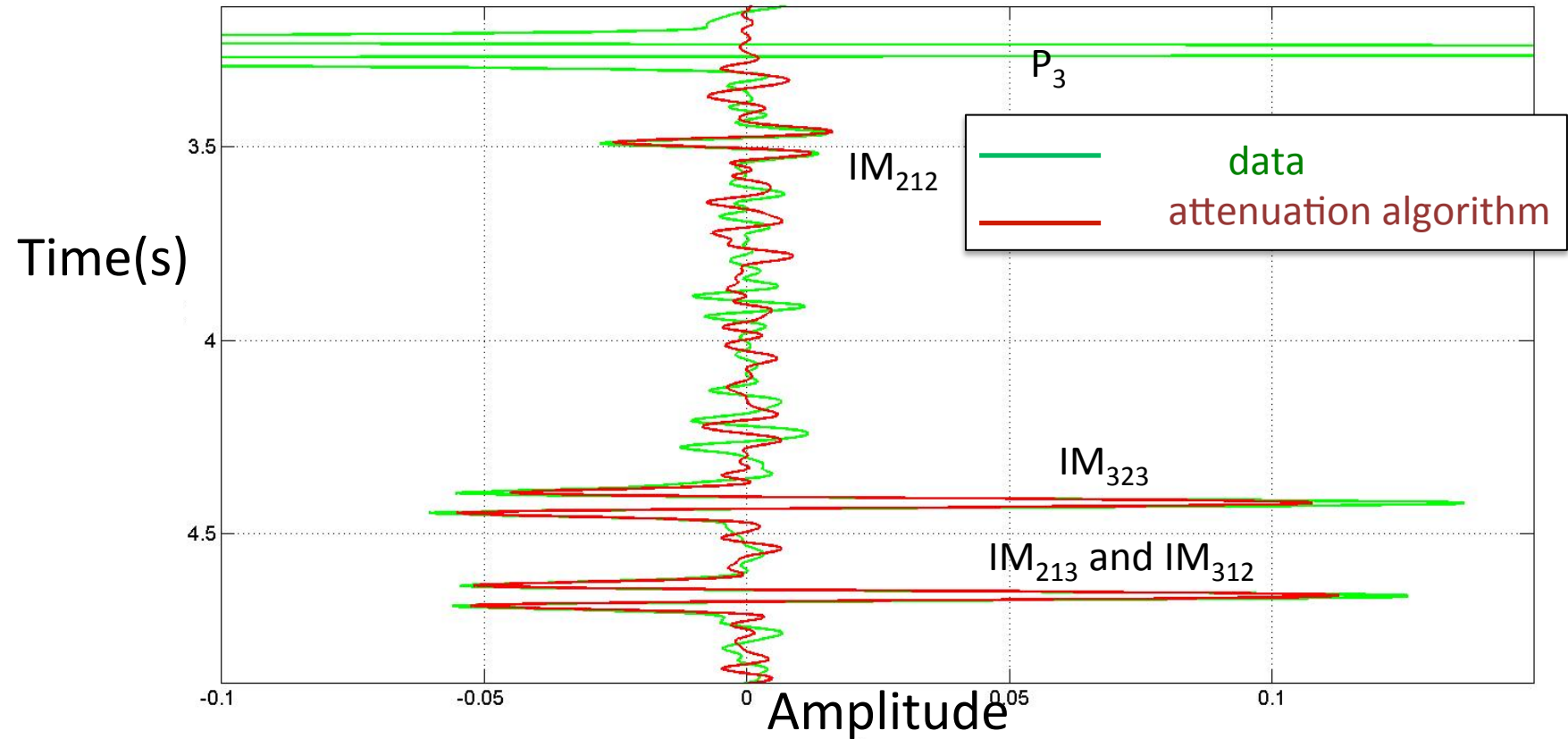
ISS internal multiple elimination

Test 1 — noise data

- A. Test for perfect data.
- B. Test for bandlimited data.
- C. Test for data with white noise.

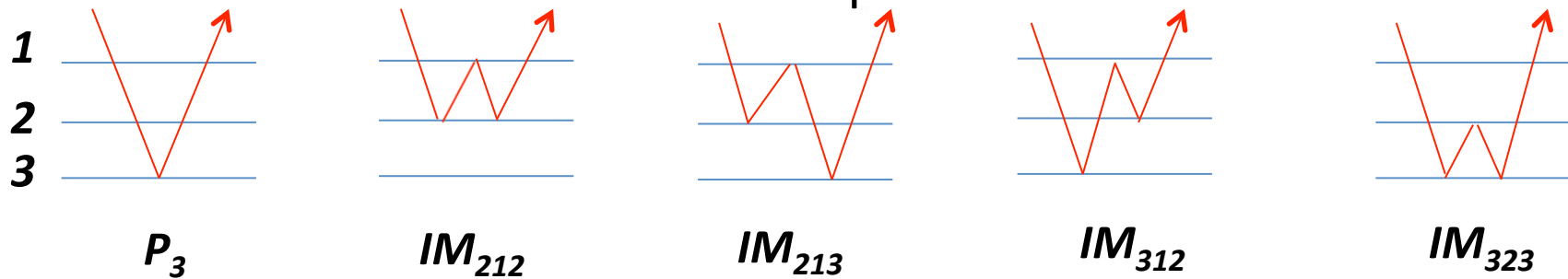
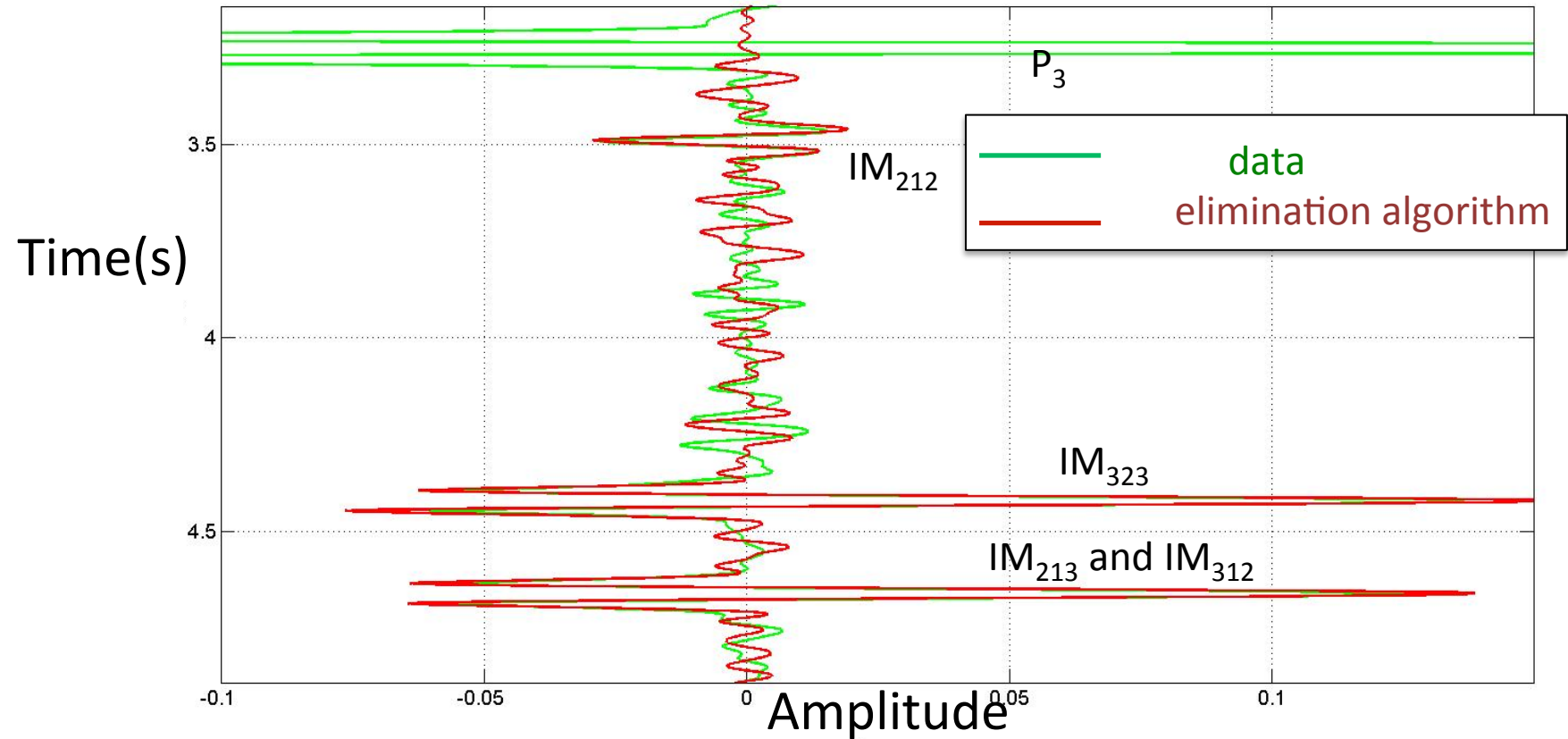
ISS internal multiple elimination

Test 1—noise data, attenuation algorithm prediction



ISS internal multiple elimination

Test 1 — noise data, elimination algorithm prediction



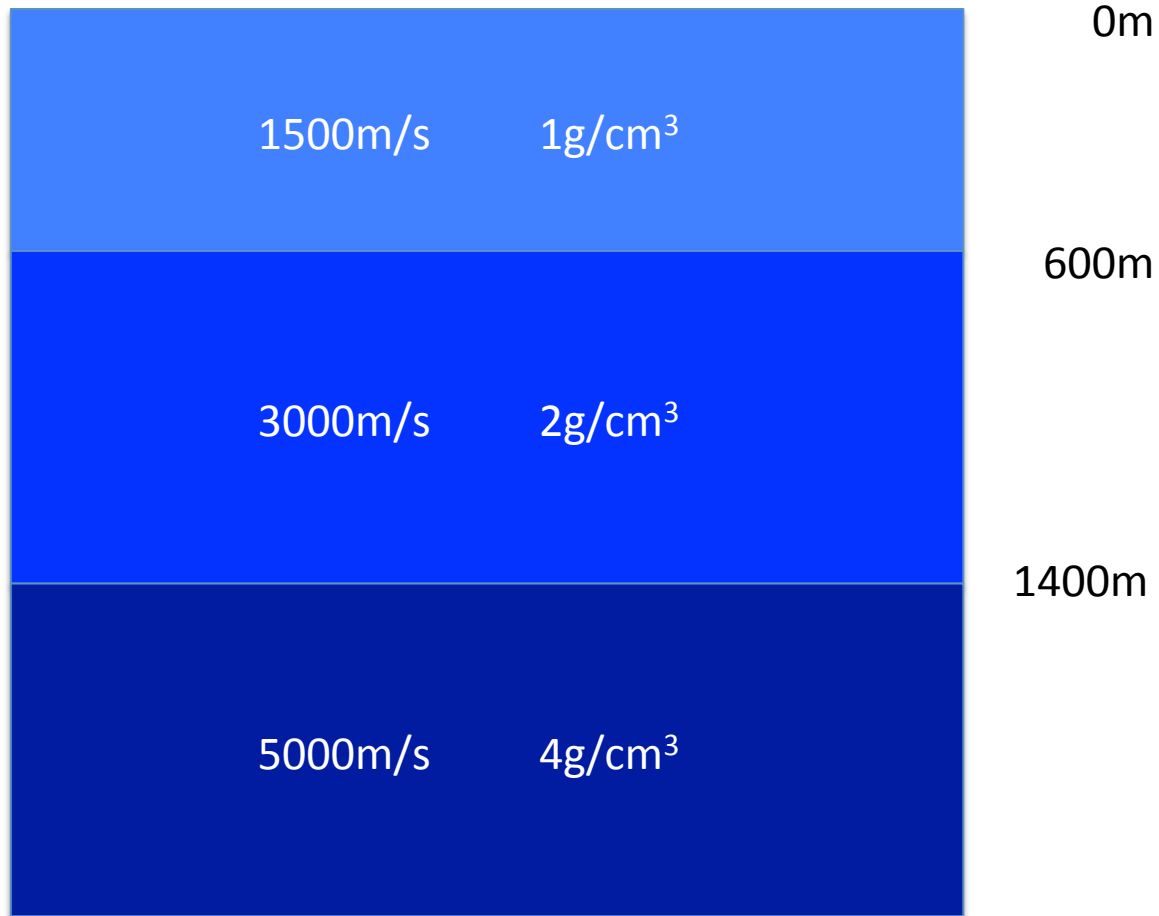
ISS internal multiple elimination

Test 2

Test 2: 1D earth offset data for 2D
sources & receivers

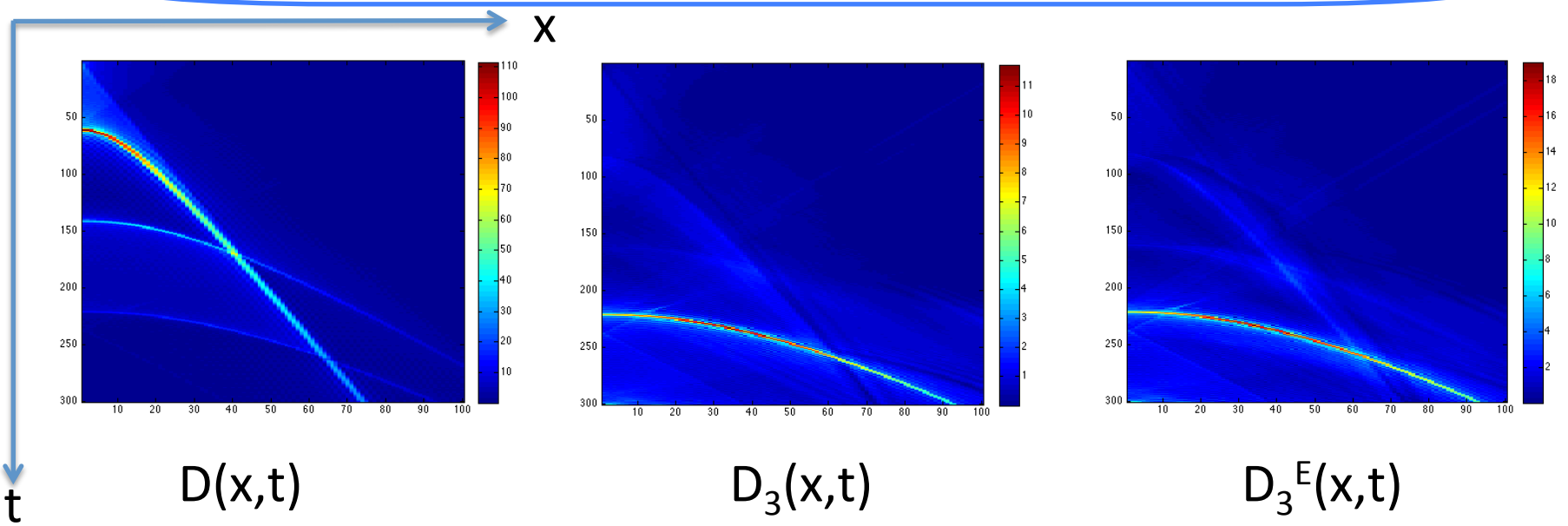
ISS internal multiple elimination

Test 2—model



ISS internal multiple elimination

Test 2 — offset = 0 m

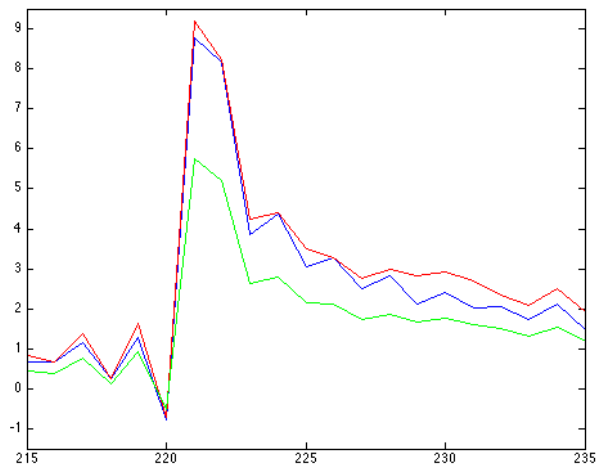


$D(x,t)$

$D_3(x,t)$

$D_3^E(x,t)$

Amplitude

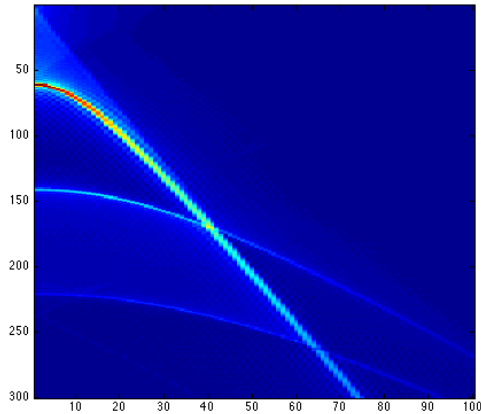


Time

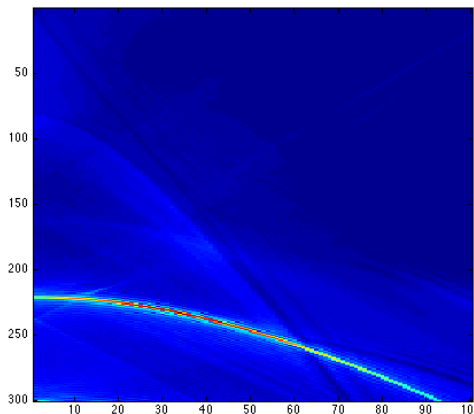
ISS internal multiple elimination

Test 2 — offset = 200 m

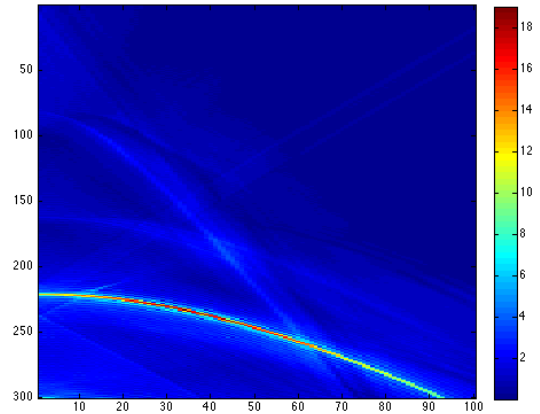
X



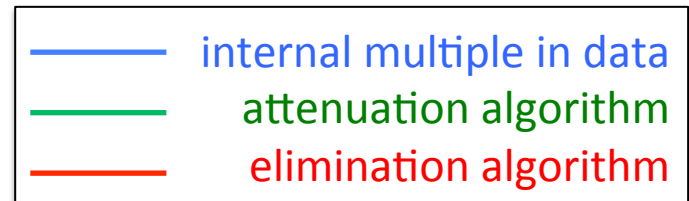
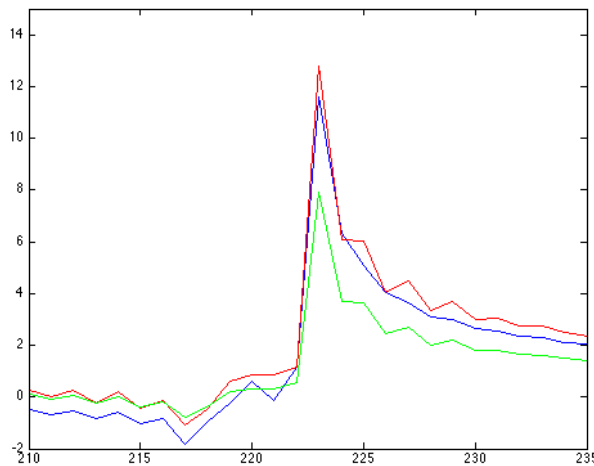
$D(x,t)$



$D_3(x,t)$



$D_3^E(x,t)$

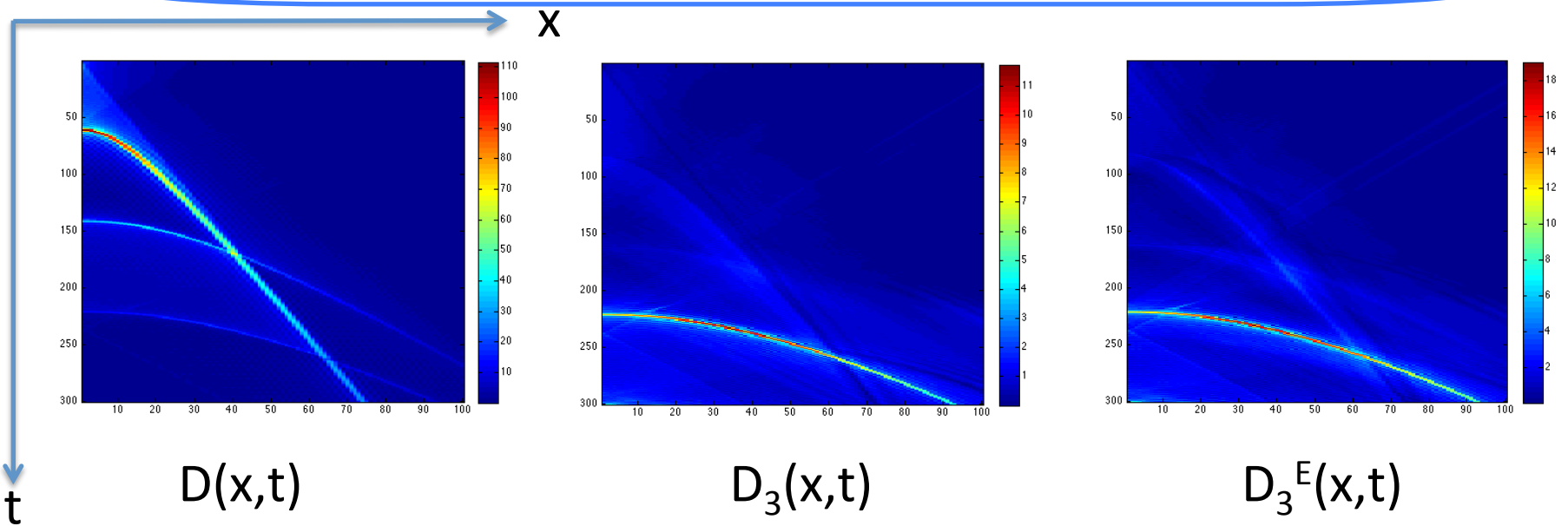


Amplitude

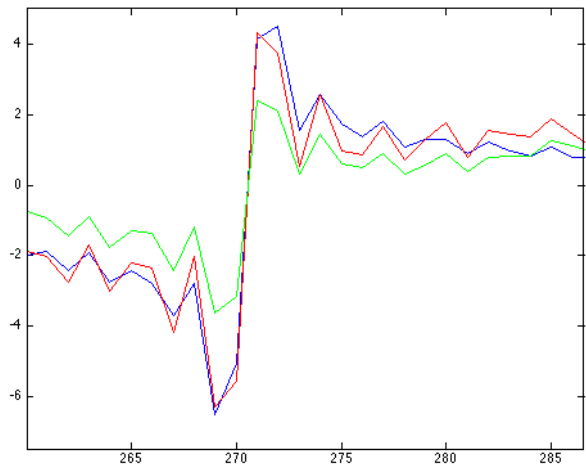
Time

ISS internal multiple elimination

Test 2 — offset = 800 m



Amplitude



Time

ISS internal multiple elimination

Flow chart of the elimination algorithm

Input: data $D(k, \omega)$

$$b_1(k, \omega) = -2iqD(k, \omega)$$

$$g(k, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}}$$

$$F[b_1(k, z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k, z')}{[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]^2 [1 - \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k, z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k, z''') e^{-iq'z'''}]}$$

$$b_E^{IM}(k, 2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k, z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k, z')] \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k, z'')$$

Output: internal multiples with correct time and amplitude $b_E^{IM}(k, 2q)$

ISS internal multiple elimination

A limitation that will be addressed in part 2

The primaries
in data b_1



Provide the elimination
capability

The internal multiples
in data b_1



will diminish the elimination
capability

ISS internal multiple elimination

A limitation that will be addressed in part 2

The primaries
in data b_1



Provide the elimination
capability

The internal multiples
in data b_1



will diminish the elimination
capability

It is the limitation of this **elimination** algorithm.
This limitation will be addressed in part 2.

ISS internal multiple elimination
discussion and future plan

***Towards internal multiple elimination:
where are we now? what are our next steps? when can you
anticipate being able to use this algorithm?***

- Current elimination algorithm is for an acoustic 1D earth for 2D sources & receivers.
- Develop elimination algorithm for an acoustic 1D earth for 3D point sources & receivers.
- Test the 3D point source 1D earth acoustic algorithm for an elastic model. If it shows added value, then we will test it on field data where the earth is close to 1D. If successful, we will distribute a documented code to sponsors.
- Develop a 3D elimination algorithm directly from terms in the inverse scattering series.

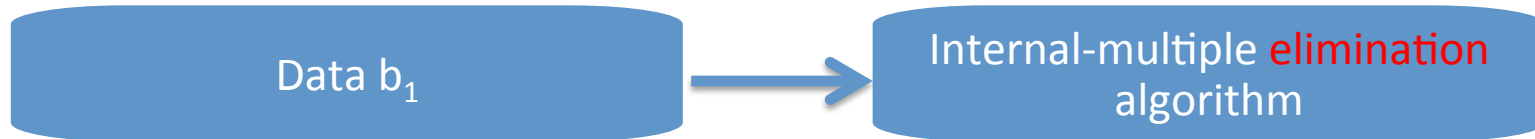


The internal-multiple *elimination* algorithm for all reflectors for 1D earth Part 2: addressing the limitations

Yanglei Zou* and Arthur B. Weglein

May 29th, 2014
Austin, TX

Addressing the limitation of the current **elimination** algorithm



Addressing the limitation of the current **elimination** algorithm

Data b_1

Internal-multiple **elimination**
algorithm

The primaries
in data b_1

Provide the elimination
capability

The internal multiples
in data b_1

will diminish the elimination
capability

Addressing the limitation of the current **elimination** algorithm

Data b_1

Internal-multiple **elimination**
algorithm

The primaries
in data b_1

Provide the elimination
capability

The internal multiples
in data b_1

will diminish the elimination
capability

It is the limitation of this **elimination** algorithm.
This limitation will be addressed in part 2.

Addressing the limitation of the current **elimination** algorithm

$$\mathbf{b}'_1 = \mathbf{b}_1 + \mathbf{b}_3$$

benefit from Chao Ma, et al (2012) Hong Liang, et al (2012)

Addressing the limitation of the current **elimination** algorithm

$$\mathbf{b}'_1 = \mathbf{b}_1 + \mathbf{b}_3$$

benefit from Chao Ma, et al (2012) Hong Liang, et al (2012)

To Address the limitation of the **elimination** algorithm, we use \mathbf{b}'_1 instead of using \mathbf{b}_1 as the input data for the ISS internal multiple elimination algorithm

Addressing the limitation of the current **elimination** algorithm

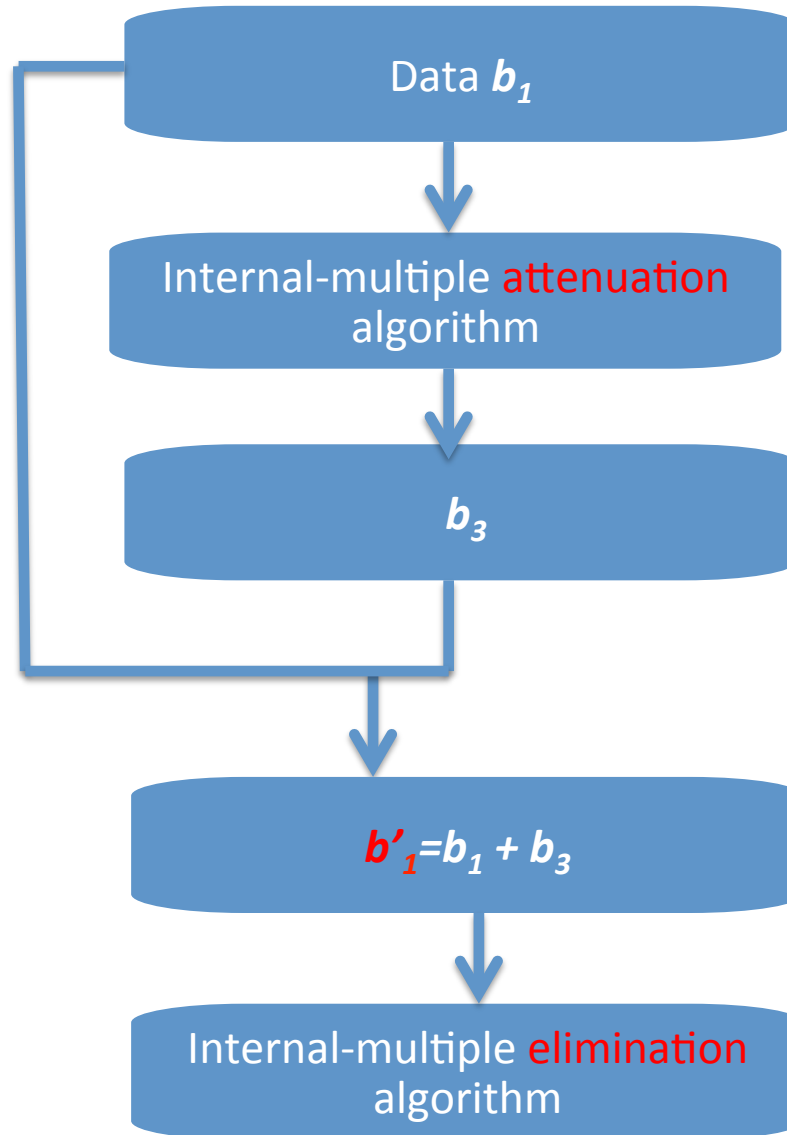
Data b_1

current elimination
algorithm



Internal-multiple **elimination**
algorithm

Addressing the limitation of the current **elimination** algorithm



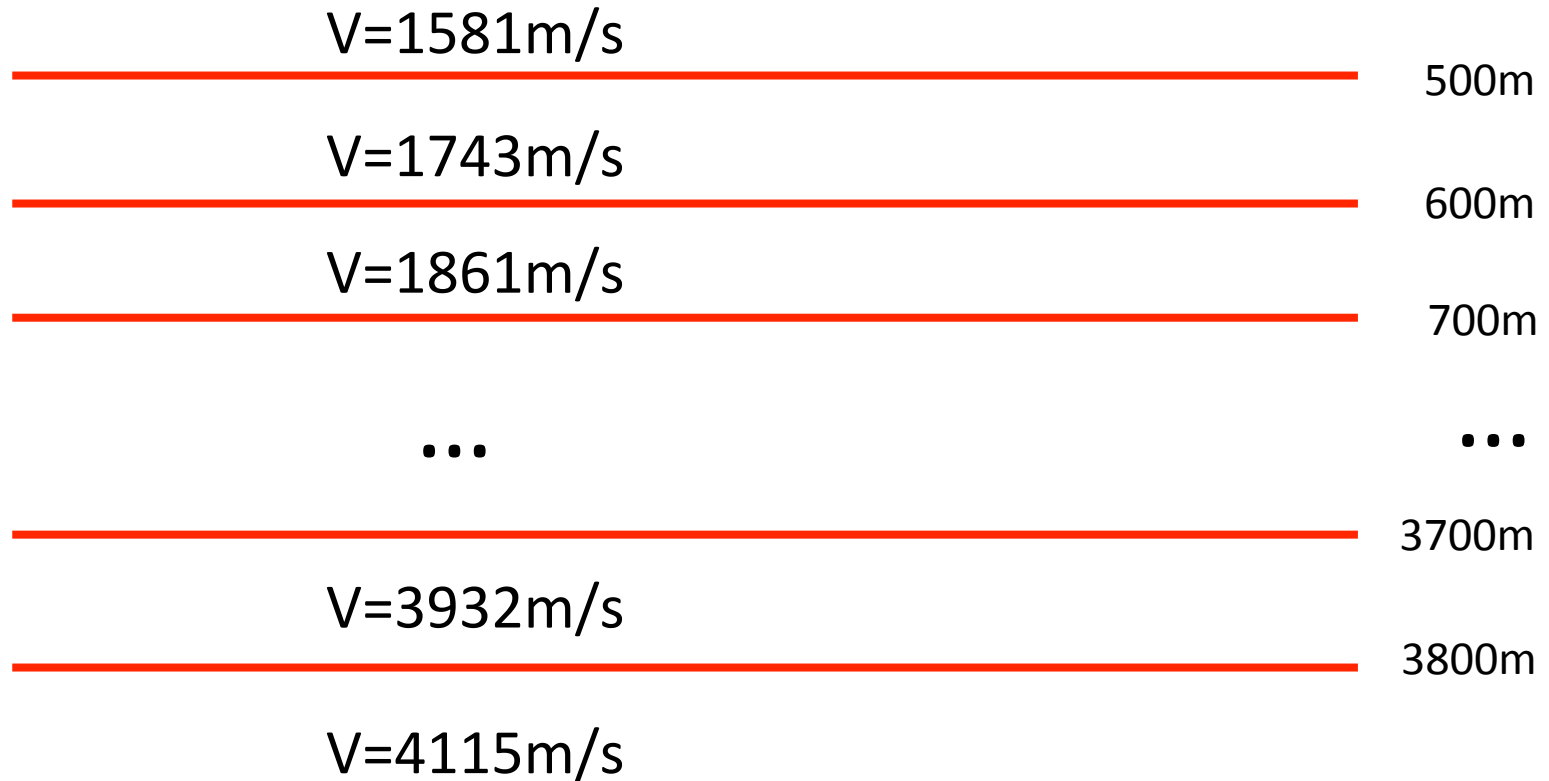
modified strategy for the
elimination algorithm

Addressing the limitation of the current **elimination** algorithm

Numerical test

1D normal incidence numerical test
based on well log velocity data

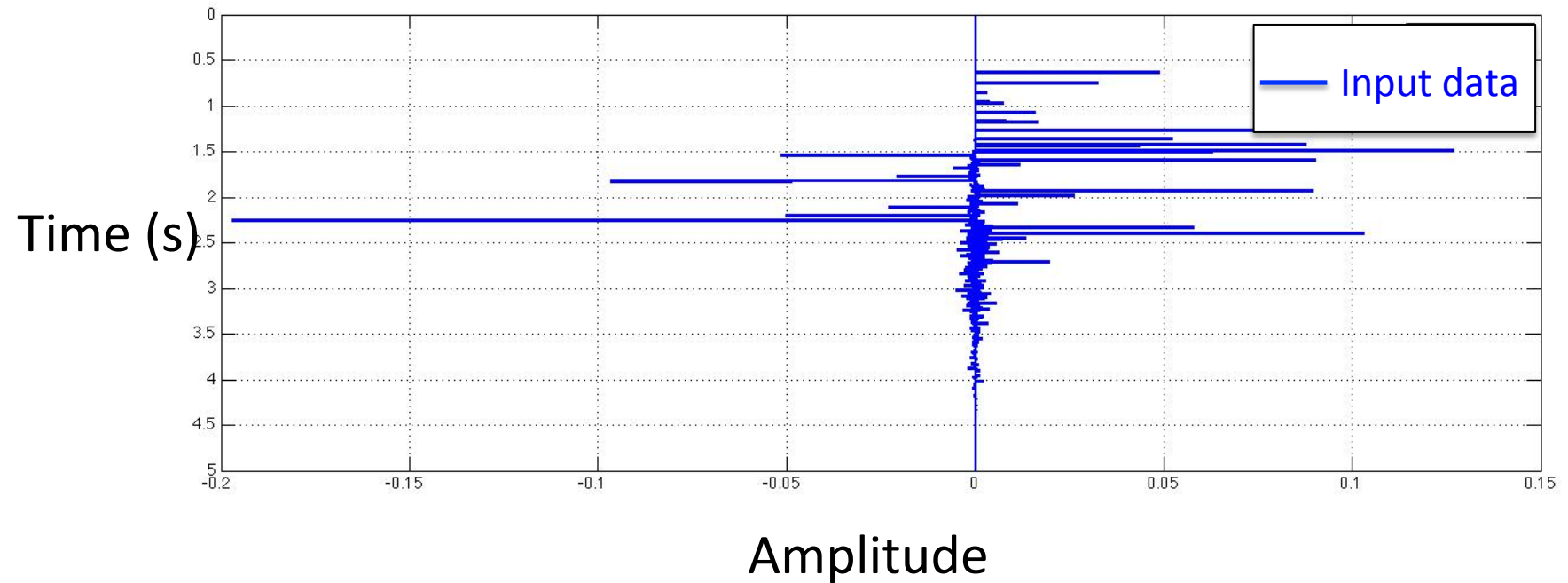
34-reflector model based on well log velocity data



(courtesy of Saudi Aramco)

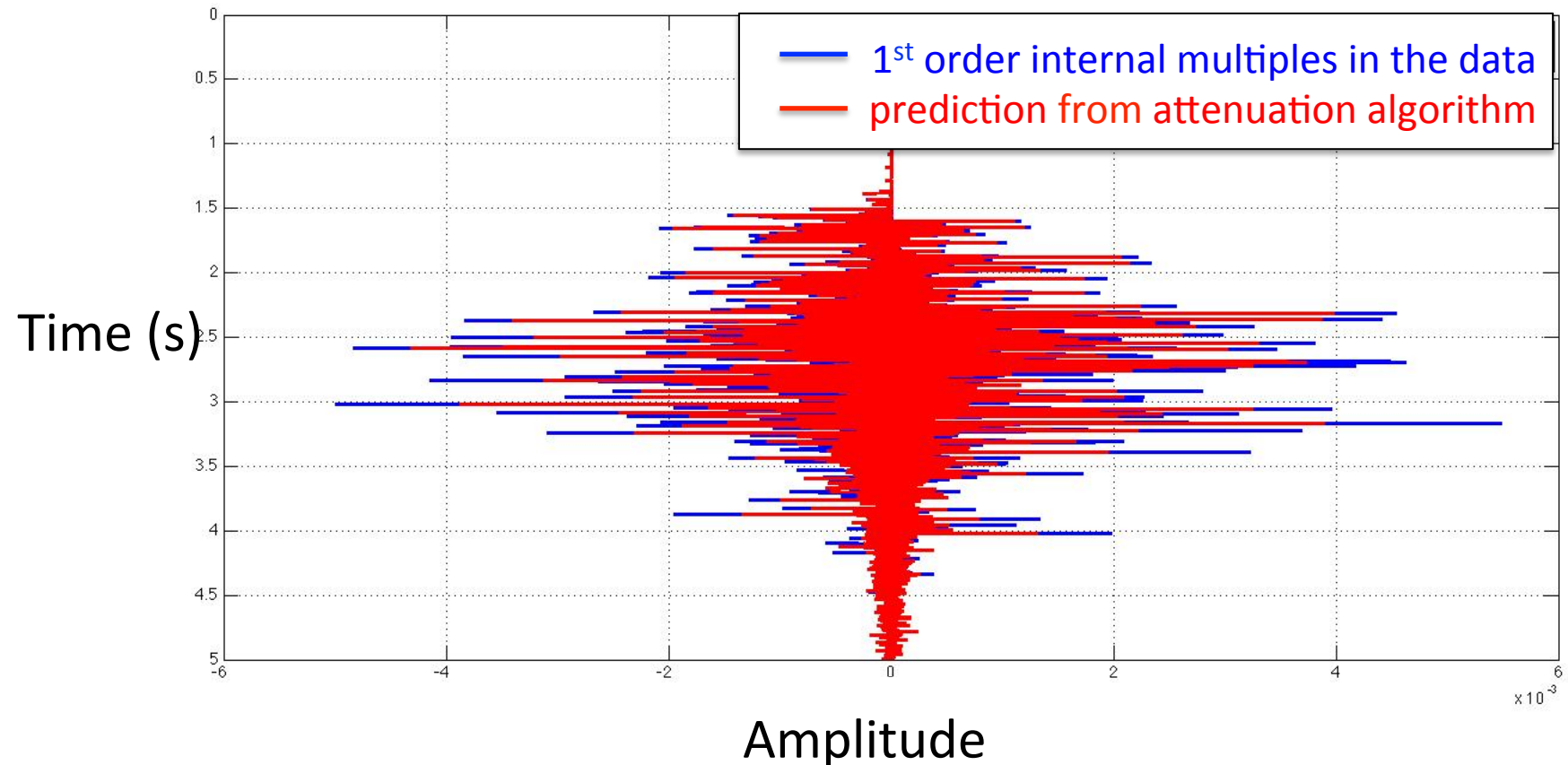
Addressing the limitation of the current **elimination** algorithm

Numerical test — input data



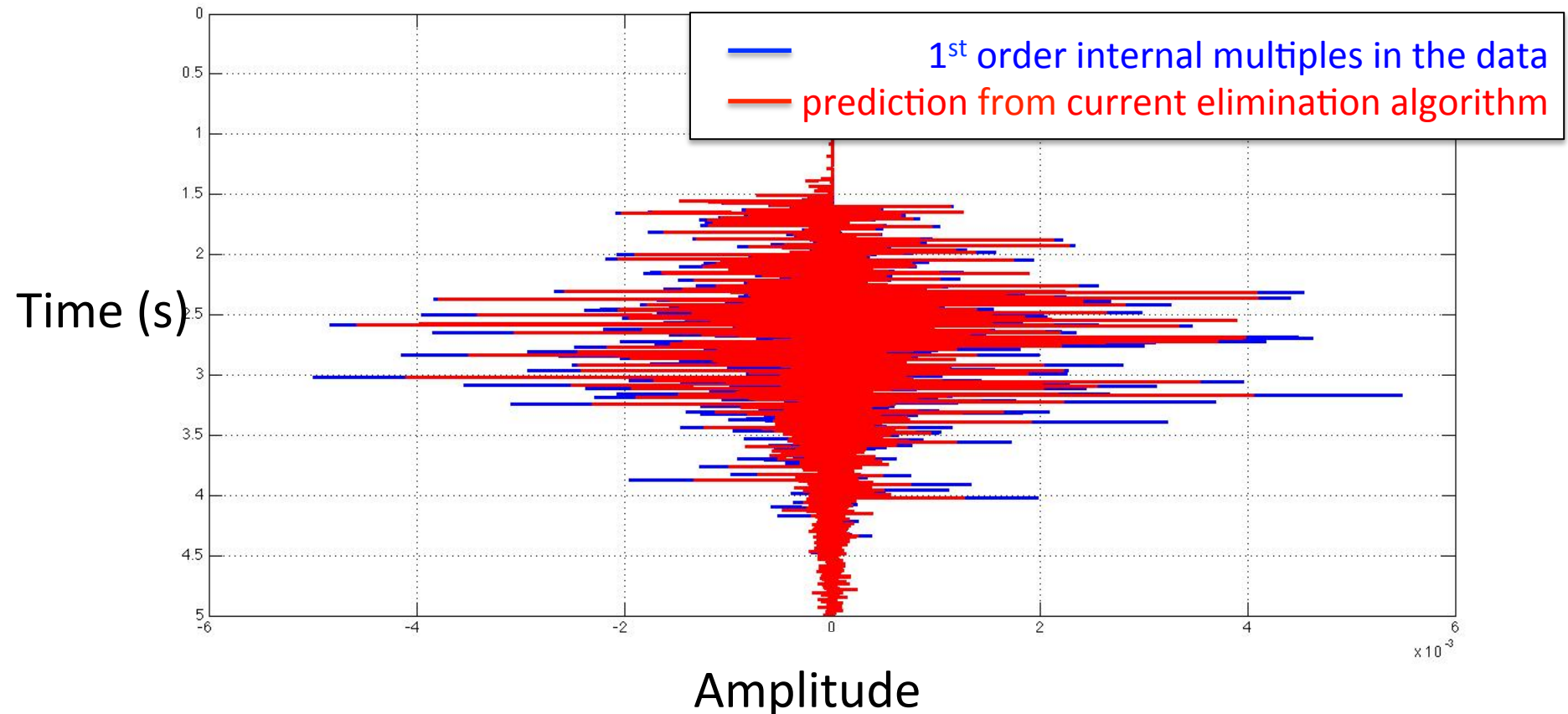
Addressing the limitation of the current **elimination** algorithm

Numerical test — prediction from attenuation algorithm

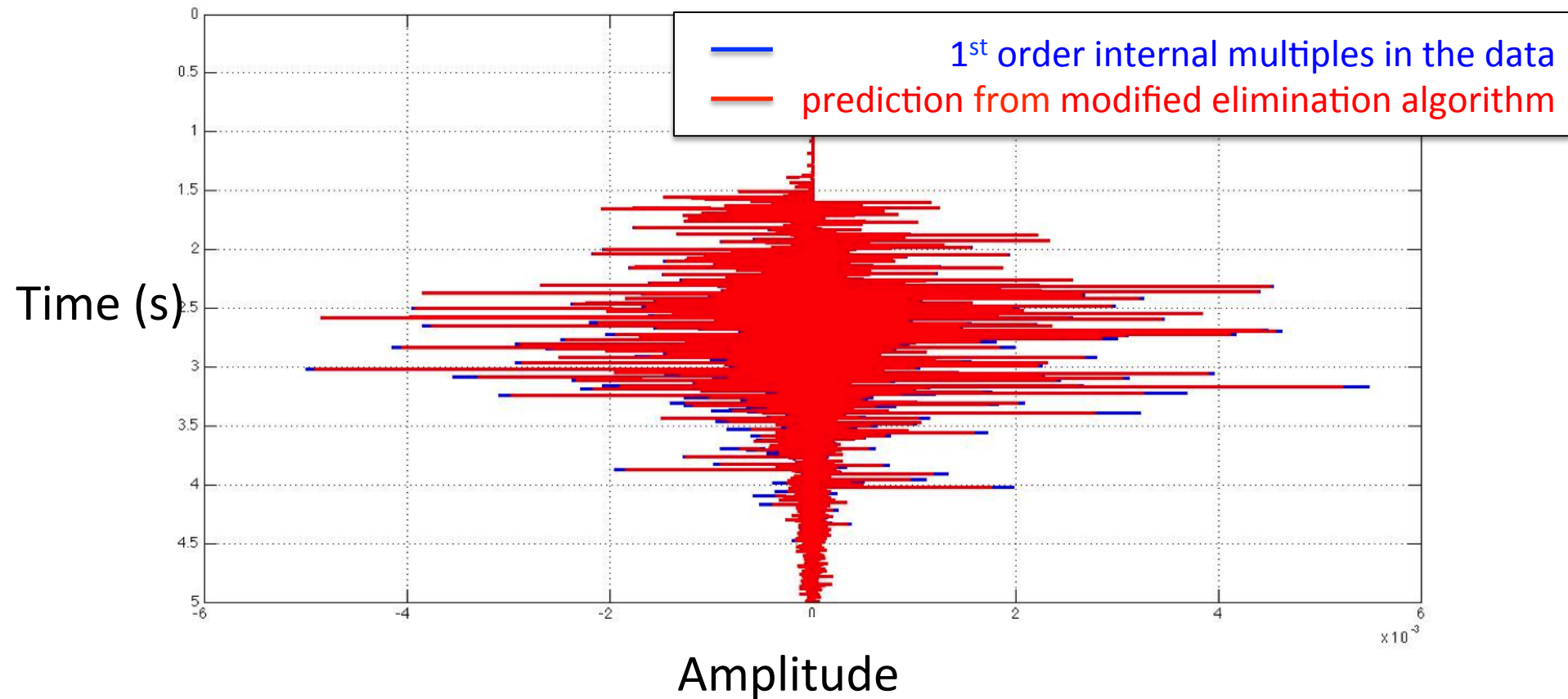


Addressing the limitation of the current **elimination** algorithm

Numerical test — prediction from current **elimination** algorithm

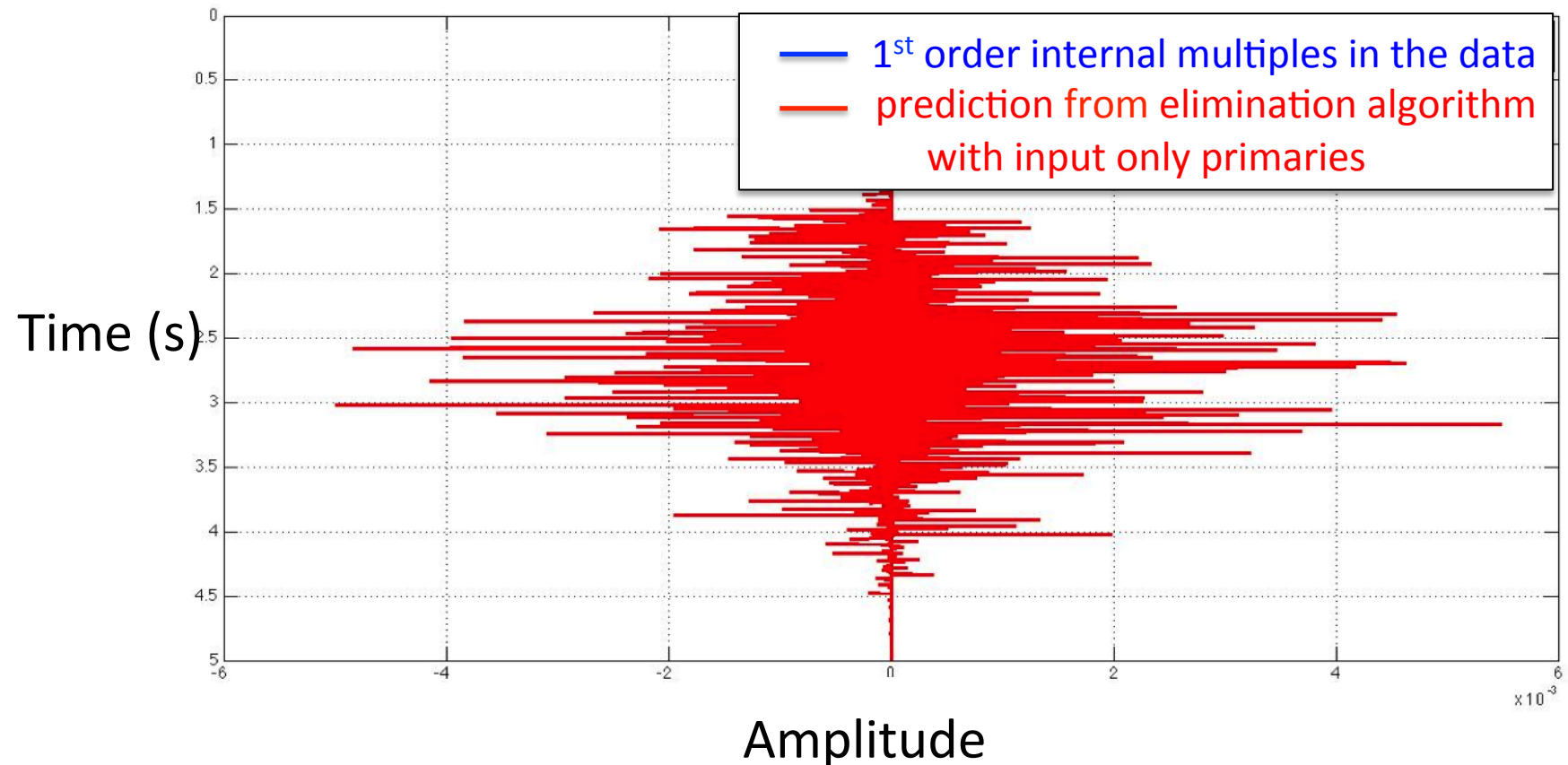


Addressing the limitation of the current **elimination** algorithm
Numerical test—prediction from modified **elimination** algorithm



Addressing the limitation of the current **elimination** algorithm

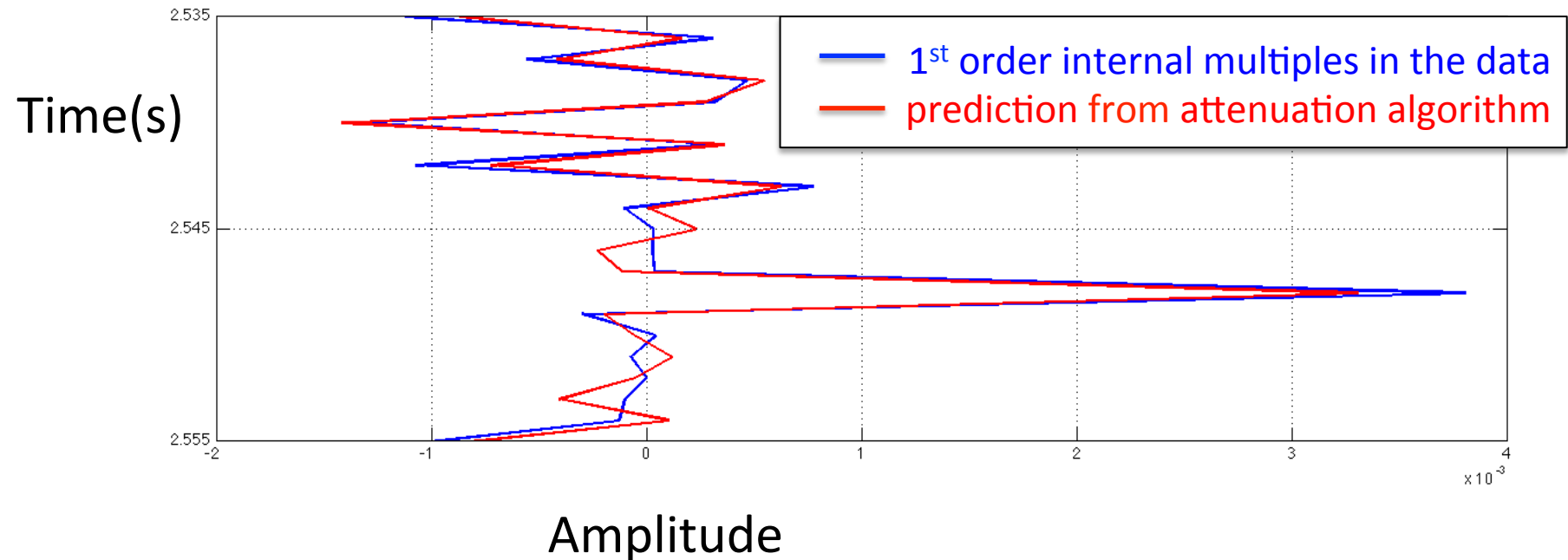
Numerical test—prediction from current **elimination** algorithm with input only primaries



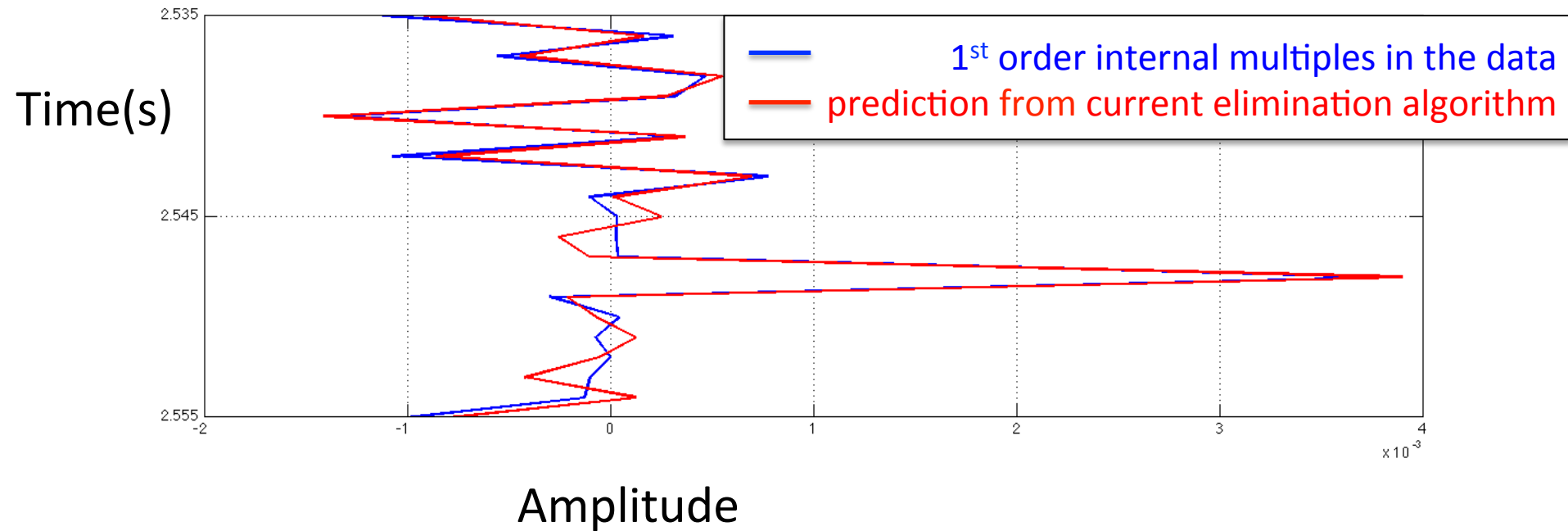
Addressing the limitation of the current **elimination** algorithm

Numerical test—prediction from attenuation algorithm

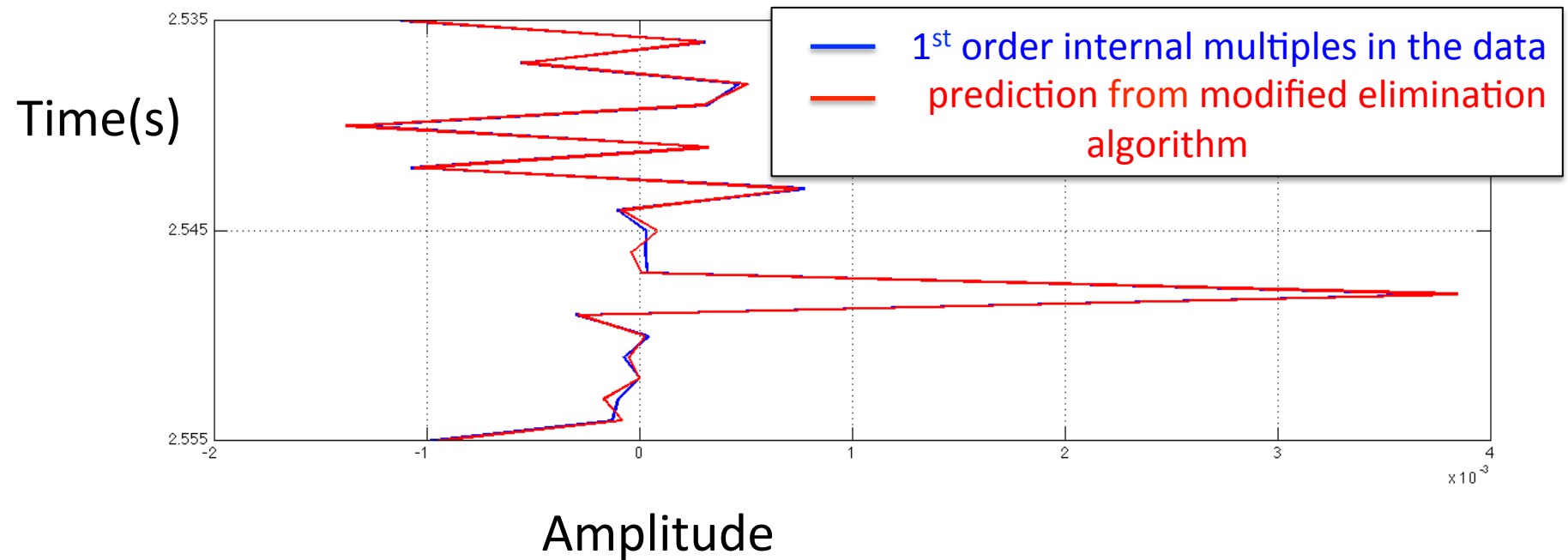
(Blow up)



Addressing the limitation of the current **elimination** algorithm
Numerical test—prediction from current **elimination** algorithm
(Blow up)

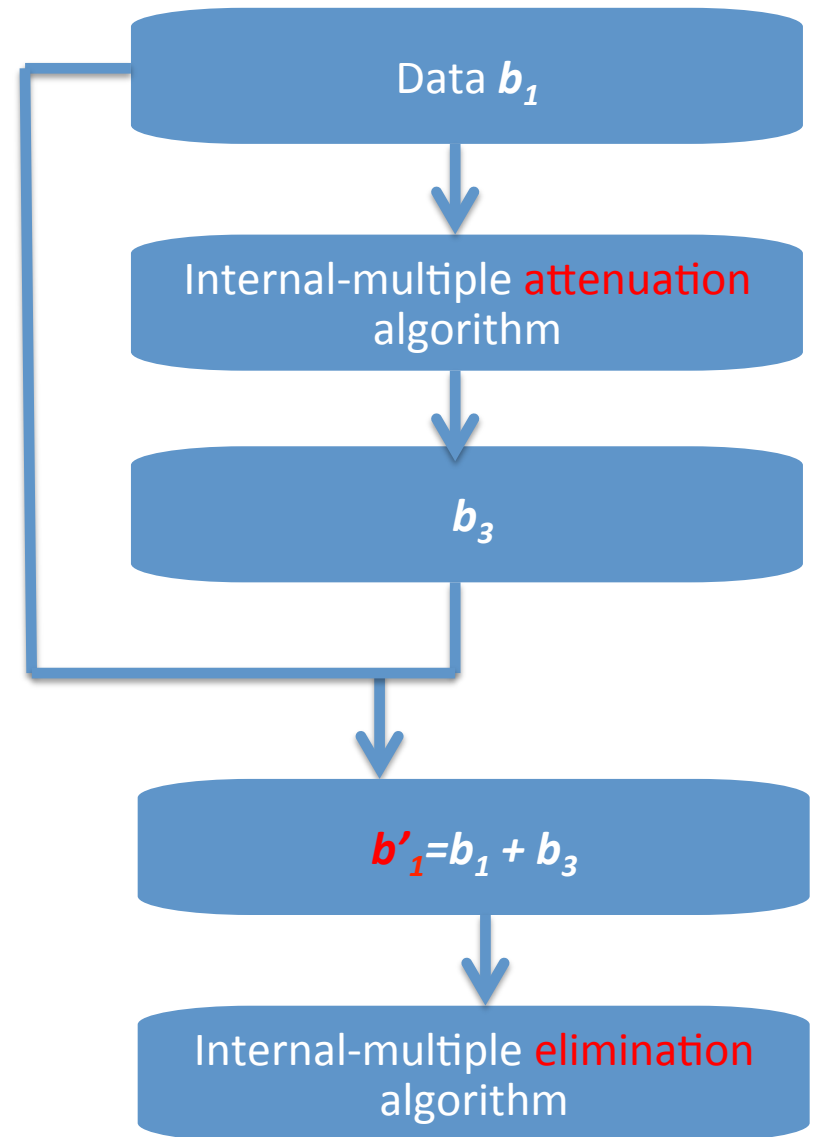


Addressing the limitation of the current **elimination** algorithm
Numerical test—prediction from modified **elimination** algorithm
(Blow up)



Addressing the limitation of the current **elimination** algorithm

the ISS internal multiple **elimination** algorithm with the limitation being addressed



Acknowledgement

- All M-OSRP sponsors for encouragement and support of this research;
- All colleagues in M-OSPR;
- Yi Luo and Saudi Arabian Oil Co. management for providing the velocity model and their encouragement.

M-OSRP

Sponsors



U.S. Federal Government Research Support



B.E.S.



C.M.G.